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# Transient analysis of quality performance in two-stage manufacturing systems with remote quality information feedback

Shihong Liu, Shichang Du*, Lifeng Xi

*Department of Industrial Engineering and Management, School of Mechanical Engineering, Shanghai Jiaotong University, Shanghai 200240, PR China*
*State Key Lab of Mechanical System and Vibration, Shanghai 200240, PR China*

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ABSTRACT

Recently modeling and analysis of product quality propagation in multi-stage manufacturing systems has received lots of attention. However, most existing results are focused on steady state performance, while transient analysis of system quality remains largely unexplored. When product changeover or scheduled maintenance happens, the system quality may undergo transients, which describe the system quality behavior before approaching the steady state at targeted levels of quality and cost. It is of practical importance to comprehensively investigate the quality performance especially during transients in order to reduce quality loss and improve product quality. In this paper, a Markov model is developed to address quality propagation in a two-stage manufacturing system with remote quality information feedback during transients. Based on the proposed mathematical model, analytical formulas for evaluating transient quality performance including the real-time product quality, settling time, and quality loss due to transients, are derived. In addition, the monotonicity properties of critical transient system characteristics and quality performance metrics are explored. The proposed method is validated with numerical data and real-world data, and the results demonstrate the effectiveness for transient quality analysis in two-stage manufacturing systems.

The notions used in this paper are described as follows.

## 1. Introduction

Flexible manufacturing systems have received substantial research in the past few decades (Zhao, Li, & Huang, 2016) and are becoming more and more important in modern manufacturing industry. For example, multiple types of engines are made in batches on the same production line. Unlike the conventional assumption that quality related issues have minimal impact, recent studies have shown that flexibility and quality are tightly coupled (Inman, Blumenfeld, Huang, & Li, 2013). In machining process, the product quality is dominated by the location precision of the flexible fixtures, and product changes will lead to quality defects introduced by errors of frequent fixture location readjustment. To deal with this, typically in practice, production is scheduled with batch policy in flexible systems to reduce product changes which may impede quality. Besides, preventive maintenance has become a prevailing trend to ensure machine reliability and product quality. After product changeover or scheduled maintenance activity happens, the system quality may undergo transients. A main reason is due to the initial condition of manufacturing system such as the flexible fixtures location readjustment errors after a new production period starts. Quality transients, which describe the system quality behavior before approaching the steady state production at targeted levels of quality and cost, are of critical practical importance. During the transients, the mean of system quality measure is not stable and can be quite different from that of the steady state, leading to quality degradation and associated quality loss. For example, three types of engine cylinder blocks (B12, B15 and N12 series) are made in batches on a flexible manufacturing line at a certain engine plant. After product changeover, the processing data of a product characteristic is recorded for successive workpieces and plotted in Fig. 1. It is shown that the processing data fluctuate rather widely for the fresh restart (the qualified rate is low) and then gradually approach the steady state. Similar scenarios can be found in automotive painting, welding and assembly systems as well (Zhao et al., 2016). However, such an issue remains largely unexplored. Few quantitative model and analytical method addressing the coupling between manufacturing system and quality propagation in terms of system transient duration are found in current literature work. Therefore, it is of critical importance to comprehensively investigate the quality performance especially during transients in order to shorten changeover time, reduce cost, and improve quality.

Most modern manufacturing systems consist of a large number of stages. In multi-stage manufacturing systems (MMSs), the variations of

* Corresponding author at: Department of Industrial Engineering and Management, School of Mechanical Engineering, Shanghai Jiaotong University, Shanghai 200240, PR China.

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**Nomenclature**

| | |
|---|---|
| $M_i$ | the $i$ th stage in MMSs |
| $g_i$ | $M_i$ is producing a good product |
| $d_i$ | $M_i$ is producing a defective product |
| $\alpha_1$ | the probability for $M_1$ to transit from state $g_1$ to state $d_1$ |
| $\beta_1$ | the probability for $M_1$ to transit from state $d_1$ to state $g_1$ |
| $\gamma_i$ | when the coming part is good, the probability for $M_i$ to transit from state $g_i$ to state $d_i$ |
| $\mu_i$ | when the coming part is good, the probability for $M_i$ to transit from state $d_i$ to state $g_i$ |
| $\eta_i$ | when the coming part is defective, the probability for $M_i$ to transit from state $g_i$ to state $d_i$ |
| $\theta_i$ | when the coming part is defective, the probability for $M_i$ to transit from state $d_i$ to state $g_i$ |
| $g_ig_{i+1}$ | $M_i$ is producing a good product and $M_{i+1}$ is also producing a good product |
| $g_id_{i+1}$ | $M_i$ is producing a good product and $M_{i+1}$ is producing a defective product |
| $d_ig_{i+1}$ | $M_i$ is producing a defective product and $M_{i+1}$ is producing a good product |
| $d_id_{i+1}$ | $M_i$ is producing a defective product and $M_{i+1}$ is also producing a defective product |
| $P(\cdot,t)$ | the probability of the system in a certain state at time $t$ |
| $X_i(t)$ | the matrix of state probabilities at time $t$ for the system with $i$ stages |
| $A_i$ | the matrix of state transition probabilities for the system with $i$ stages |
| $P(g_i,t)$ | the probability of producing good product for the system with $i$ stages at time $t$ |
| $\lambda_2$ | the second largest eigenvalue (SLE) of the state transition probabilities matrix, which characterizes the duration of system quality transients |
| $P(g_i)_{SS}$ | the steady-state probability to produce good product for the system with $i$ stages |
| $\Phi_2$ | the pre-exponential coefficient (PEC) corresponding to SLE, which characterizes the impact that the SLE has on the system transients of quality performance |
| $t_S$ | the settling time of the system quality performance to reach the steady state |
| $L_Q$ | the quality loss due to system transients |

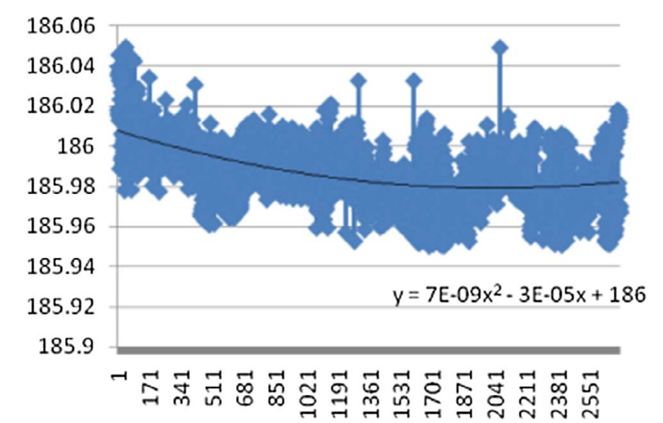



**Fig. 1.** The processing data of a product characteristic for successive workpieces after system restarts.

final quality are the accumulation of variations introduced and propagated from all stages. Among the enormous research on modeling and analysis of MMSs for quality, analytical methods are presented based on fundamental physical laws. The pioneering work of Jin and Shi (1999) proposed the most popular analytical method of state space model, which links engineering knowledge of variation sources with final quality measurement data. Later, the state space model is extended into three dimensional assembly systems (Huang, Lin, Bezdecny, Kong, & Ceglarek, 2007a; Huang, Lin, Kong, & Ceglarek, 2007b) and machining systems (Abellan-Nebot, Liu, Subirón, & Shi, 2012; Du, Yao, & Huang, 2015c; Du, Yao, Huang, & Wang, 2015b). A complete review of related work is provided by Shi (2006) and Shi and Zhou (2009). Typically the state space model depends on complicated kinematics of manufacturing process, or is only applicable to dimensional errors which limits its further application (Ju, Li, Xiao, Huang, & Biller, 2014).

In another direction, there has been an increasing trend of exploring the coupling between manufacturing system and product quality using Markov analytic models. In Inman et al. (2013), review of related papers and empirical evidence concluded that manufacturing system design has a significant impact on quality and several research opportunities were presented from the automotive industry perspective. Since then, the interaction between manufacturing system and quality has become a focus of research. Wang, Li, Arinez, and Biller (2012, 2013) developed Markov models to quantify the probability of good parts, investigated the nonmonotonic properties, and introduced indicators for the quality improvability. A serial production line with deterministic service durations and random setups was modeled as a Markov chain in semiconductor manufacturing (Kim & Morrison, 2015). A Markovian approach was developed to model the effects of maintenance on wind turbine components at components lifecycle phases (Ossai, Boswell, & Davies, 2016). Zhong, Li, Bain, and Musa (2016a) introduced a Markov model to study e-visits in primary care clinics. The results show that the first come first serve policy typically leads to the best performance. Xie, Li, Swartz, Dong, and DePriest (2016) presented a Markov chain to describe the ward patient status and analyze the patient rescue processes, which are characterized by the transitions between different patient states. A system-theoretic method based on Markov model was presented to address the limited availability of care providers in a mammography testing center in Zhong, Li, Ertl, Hassemer, and Fiedler (2016b). Applications of Markov modeling approach also include automotive paint shops (Ju, Li, Xiao, & Arinez, 2013), a furniture assembly system (Zhao & Li, 2014), battery manufacturing (Ju et al., 2014), patient care delivery (Wang, Zhong, Li, & Howard, 2014), stochastic inventory control policy (Zhu, Liu, & Chen, 2015), surgical work flow disruptions (Shao et al., 2015), condition-based maintenance policy (Tang, Yu, Chen, & Makis, 2015), scrap counts reduction in semiconductor systems (Wu, Chien, Chuang, & Cheng, 2016), energy conversion equipment degradation (Zhou, Yu, Zhang, & Weng, 2016).

Despite of these efforts, note that most of the current research addressing product quality and manufacturing systems are concerned with steady-state analysis, while the behavior of system quality during transients is still in need for further exploration. Recently, there has been a rising effort devoted to transient analysis of manufacturing systems in terms of throughput analysis. Among the publications available on transient throughput performance, Zhang, Wang, Arinez, and Biller (2013) studied the transient throughput properties of production lines based on Markov model in the framework of finite buffers and Bernoulli reliability machines. Later more extending studies include multi-stage Bernoulli machines (Wang & Li, 2015), multi-stage geometric machines (Chen, Wang, Zhang, Arinez, & Xiao, 2016), assembly systems (Jia, Zhang, Arinez, & Xiao, 2015a), batch-based production lines (Jia, Zhang, Arinez, & Xiao, 2014), finite production run-based serial lines (Jia, Zhang, Chen, Arinez, & Xiao, 2016b), closed production lines (Jia & Zhang, 2017). Applications of transient throughput analysis in Bernoulli lines are reported in Wang, Hu, and Li

(2010), Chen, Zhang, Arinez, and Biller (2013), Jia, Zhang, Arinez, and Xiao (2015b, 2016a), Ju, Li, and Horst (2017).

Although considerable research has been devoted to steady-state quality performance in manufacturing systems, to the best of our knowledge, there is no research paper focused on developing analytical methods to evaluate the quality propagation in manufacturing systems with remote quality information feedback (RQIF) during transients. In manufacturing systems with RQIF, defective products from upstream stages will not go out of the system until the final stage and may be corrected by downstream stages. Actually in real manufacturing systems, there do exist the condition where a part with dissatisfactory quality becomes good after processed by downstream stages. Taking a hole with dimension requirement $10^{+0.04}_{-0.04}$ (mm) for instance, when after rough machining, its dimension is 9.7 (mm) which is dissatisfactory, then it can be corrected to $10^{+0.04}_{-0.04}$ (mm) by the downstream finish stage. In other words, the coming parts may be good or defective for each stage before being processed and there exist both quality degradation and quality correction. Therefore, developing a method to reflect these characteristics and to investigate the quality propagation in such systems during transients is of importance. This paper is intended to contribute to this end. The main contribution of this paper is in developing a Markov model to evaluate the dynamics of quality performance in a manufacturing system with RQIF during transients. Closed formulas to describe the transient quality are derived and structural properties of system operational parameters with respect to quality are investigated.

The rest of this paper is organized as follows. Section 2 introduces problem assumptions and formulates a Markov model to analyze quality propagation in two-stage manufacturing system with RQIF during transients. Analytical expressions to evaluate the evolution of system quality performance are derived. In Section 3, transient quality characteristics and the monotonicity properties are analyzed. In Section 4, the settling time and an approximation are investigated. In Section 5, quality loss due to transients is explored and guidance for continuous improvement is presented. A case study at engine manufacturing plant is conducted to verify the proposed method in Section 6. Finally, conclusions are formulated in Section 7.

## 2. Modeling of manufacturing systems

### 2.1. Assumptions and problem formulation

In reality due to resource constraints, manufacturing systems with RQIF are very common (Montgomery, 2009). RQIF represents the situation where most but not all operations are reliable in quality and the quality defects are only inspected and identified at the end of the production line. Such systems can be found in assembly systems (Zantek, Wright, & Plante, 2006), semiconductor manufacturing (Kim & Gershwin, 2008), engine manufacturing (Du & Xi, 2012), and aircraft horizontal stabilizer assembly (Du & Lv, 2013; Du, Lv, & Xi, 2012).

The following assumptions 1–6 address manufacturing systems with RQIF, system state transition, inspection and quality characteristics.

1. The manufacturing system consists of $n$ stages and an inspection station which is at the end of the manufacturing system.
2. The time axis is slotted with slot duration $\tau$ equals to the cycle time of the machines. Only the working or production period of the system is considered. Machine breakdowns are not considered.
3. The quality of the product manufactured by stage $M_i(i \geqslant 2)$ relies on both the state of stage $M_i$ and the quality of the coming part from upstream stage $M_{i-1}$. There exist not only quality degradation but also quality correction in the system. The product quality may get worse or better after processed by a certain stage.
4. In terms of the state of stage $M_i$, define that the stage $M_i(i = 1,2,...,n)$ is in a good state $g_i$ or a defective state $d_i$ if it is producing a product with good quality or defective quality at time $t$.

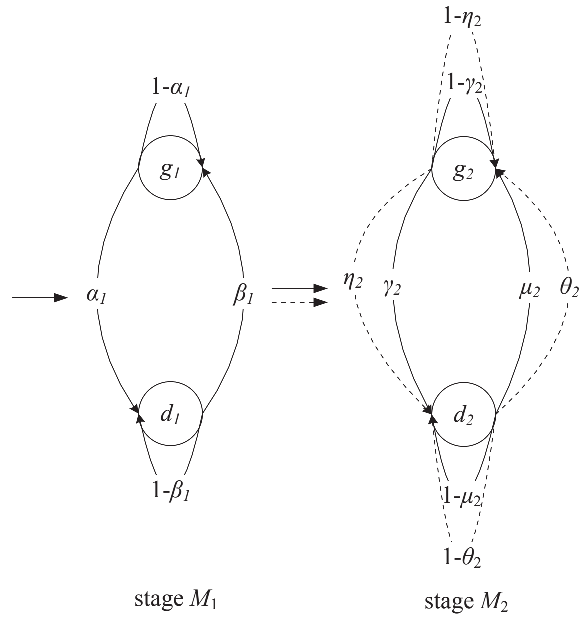



**Fig. 2.** State transition diagrams of two-stage manufacturing systems.

5. In terms of the quality of the coming part for stage $M_i(i \geqslant 2)$ at time $t$, it depends on the state of product from upstream stage $M_{i-1}$ at time $(t-1)$. The good state $g_{i-1}$ or defective state $d_{i-1}$ of stage $M_{i-1}$ means good product or defective product after processed by stage $M_{i-1}$ at time $(t-1)$, which is good or defective coming part for $M_i$ at time $t$, respectively.
6. The state of $M_1$ is not affected by the state of $M_2$. When $M_1$ is in good state $g_1$, it has probability $\alpha_1$ to transit to defective state $d_1$ and probability $(1-\alpha_1)$ to good state $g_1$. When $M_1$ is in defective state $d_1$, it has probability $\beta_1$ to transit to good state $g_1$ and probability $(1-\beta_1)$ to defective state $d_1$ (see Fig. 2).

With good coming parts, when $M_i(i \geqslant 2)$ is in good state $g_i$, it has probability $\gamma_i$ to transit to defective state $d_i$ and probability $(1-\gamma_i)$ to good state $g_i$. When $M_i$ is in defective state $d_i$, it has probability $\mu_i$ to transit to good state $g_i$ and probability $(1-\mu_i)$ to defective state $d_i$ (see Fig. 2).

With defective coming parts, when $M_i(i \geqslant 2)$ is in good state $g_i$, it has probability $\eta_i$ to transit to defective state $d_i$ and probability $(1-\eta_i)$ to good state $g_i$. When $M_i$ is in defective state $d_i$, it has probability $\theta_i$ to transit to good state $g_i$ and probability $(1-\theta_i)$ to defective state $d_i$ (see Fig. 2).

The state transition diagrams of two-stage manufacturing systems is shown in Fig. 2. Between the stages, the solid line with arrow represents good coming parts, and the dashed line with arrow represents defective coming parts.

Note that $\alpha_1,\gamma_i,\eta_i(i \geqslant 2)$ are referred as quality failure probabilities and $\beta_1,\mu_i,\theta_i(i \geqslant 2)$ as quality repair probabilities. In this paper, we focus on two-stage manufacturing systems with RQIF (see Fig. 3). Multi-stage manufacturing systems with RQIF is more complicated and will be studied in the future.

The transition probabilities of the manufacturing system can be estimated based on statistical analysis of historical processing data. The steps are as follows. We first keep records of the product quality before and after each stage and mark them as good or defective. For a certain

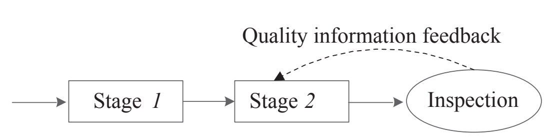



**Fig. 3.** A two-stage manufacturing system with remote quality information feedback.

part $j$ which is processed by stage $M_{i-1}(i \geqslant 2)$, it can be either good or defective coming part for the downstream stage $M_i$. And the previous part ($j$−1) after processed by $M_i$ may also be good or defective. After part $j$ is processed by $M_i$, in terms of a good or defective coming part, there exist four possible statuses for $M_i$, respectively.

(1) The previous part ($j$−1) after processed by $M_i$ is good and part $j$ is also good;
(2) The previous part ($j$−1) after processed by $M_i$ is good but part $j$ is defective;
(3) The previous part ($j$−1) after processed by $M_i$ is defective but part $j$ is good;
(4) The previous part ($j$−1) after processed by $M_i$ is defective and part $j$ is also defective.

When the coming part is good, the proportion of statue (2) represents transition probability $\alpha_1$ of $M_1$ or $\gamma_i$ of $M_i(i \geqslant 2)$. And the proportion of statue (3) represents transition probability $\beta_1$ of $M_1$ or $\mu_i$ of $M_i(i \geqslant 2)$. When the coming part is defective, the proportions of (2) and (3) would be taken as $\eta_i$ and $\theta_i$, respectively. By implementing the steps, the transition probabilities data necessary are estimated based on historical processing data analysis.

The steady state performance of manufacturing systems described by assumptions 1–6 has been explored in Du, Xu, Huang, and Yao (2015a). This paper addresses their transient quality. Thus the problem to be addressed is: under the above assumptions, develop analytical a model that describes the transient quality of two-stage manufacturing systems with RQIF as a function of system parameters, and to develop analytical methods for their quality performance evaluation during transients.

### 2.2. Mathematical model

For a two-stage manufacturing system, it has the following four quality states at a certain time $t$:(1) state $g_1g_2$ which means that both $M_1$ and $M_2$ are producing good products; (2) state $g_1d_2$ which means that $M_1$ is producing good product while $M_2$ is producing defective one; (3) state $d_1g_2$ which means that $M_1$ is producing defective product while $M_2$ is producing good one; (4) state $d_1d_2$ which means that both $M_1$ and $M_2$ are producing defective products.

Under assumptions 1–6, the two-stage manufacturing system with RQIF is characterized by an ergodic Markov chain with the four states described above. The states of the Markov chain at time $t$ in matrix form are denoted as

$$X_2(t) = [P(g_1g_2,t) \quad P(g_1d_2,t) \quad P(d_1g_2,t) \quad P(d_1d_2,t)]^T \tag{1}$$

The system transits among these four states with certain transition probabilities. To calculate the state transition probabilities between the four states, take the transition from state $(g_1g_2,t)$ to state $(d_1g_2,t+1)$ as an example. This transition means that: (1) $M_1$ produces good product and passes it to $M_2$, then $M_1$ transits from producing good product to producing defective one with probability $\alpha_1$ (see assumption 6); (2) with good coming part, $M_2$ maintains good state $g_2$ with probability $(1-\gamma_2)$ (see assumption 6). The transition probability from state $(g_1g_2,t)$ to $(d_1g_2,t+1)$ can be calculated as the product of the two probabilities, $\alpha_1(1-\gamma_2)$. Similarly, all the other transition probabilities can be obtained. And put these transition probabilities in matrix form, we have the state transition probabilities matrix,

$$A_2 = \begin{bmatrix} (1-\alpha_1)(1-\gamma_2) & (1-\alpha_1)\mu_2 & \beta_1(1-\eta_2) & \beta_1\theta_2 \\ (1-\alpha_1)\gamma_2 & (1-\alpha_1)(1-\mu_2) & \beta_1\eta_2 & \beta_1(1-\theta_2) \\ \alpha_1(1-\gamma_2) & \alpha_1\mu_2 & (1-\beta_1)(1-\eta_2) & (1-\beta_1)\theta_2 \\ \alpha_1\gamma_2 & \alpha_1(1-\mu_2) & (1-\beta_1)\eta_2 & (1-\beta_1)(1-\theta_2) \end{bmatrix} \tag{2}$$

To illustrate the matrix $A_2$, take state probability $P(g_1g_2,t+1)$ for example, the system can transit to state $P(g_1g_2,t+1)$ from state $P(g_1g_2,t)$, $P(g_1d_2,t)$, $P(d_1g_2,t)$, $P(d_1d_2,t)$ with certain transition probability, respectively. We have

$$\begin{aligned} P(g_1g_2,t+1) &= P(g_1g_2,t+1|g_1g_2,t)P(g_1g_2,t) + P(g_1g_2,t+1|g_1d_2,t)P(g_1d_2,t) \\ &+ P(g_1g_2,t+1|d_1g_2,t)P(d_1g_2,t) + P(g_1g_2,t+1|d_1d_2,t)P(d_1d_2,t) \\ &= (1-\alpha_1)(1-\gamma_2)P(g_1g_2,t) + (1-\alpha_1)\mu_2P(g_1d_2,t) \\ &+ \beta_1(1-\eta_2)P(d_1g_2,t) + \beta_1\theta_2P(d_1d_2,t) \end{aligned}$$

To calculate the final quality of the product, calculate the probability that $M_2$ is in state $g_2$ of producing good quality product. Denote $P(g_2,t)$ as the probability of producing good product of the system and it follows that,

$$P(g_2,t) = P(g_1g_2,t) + P(d_1g_2,t) \tag{3}$$

Similarly, the probability to produce a defective part $P(d_2,t)$ is

$$P(d_2,t) = P(g_1d_2,t) + P(d_1d_2,t) \tag{4}$$

The evolution of $X_2(t)$ can be described by the following constrained linear equation:

$$X_2(t+1) = A_2X_2(t) \tag{5}$$

$$P(g_1g_2,t) + P(g_1d_2,t) + P(d_1g_2,t) + P(d_1d_2,t) = 1 \tag{6}$$

And the evolution of $P(g_2,t)$ and $P(d_2,t)$ is

$$y_2(t) = \begin{bmatrix} P(g_2,t) \\ P(d_2,t) \end{bmatrix} = CX_2(t) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} X_2(t) \tag{7}$$

Eqs. (5)–(7) describe the transients of the system and the transient quality performance measures. Actually the results in Du et al. (2015a) for steady-state quality analysis can be regarded as a special case when $t \to \infty$. In this paper, we derive methods to analyze these performance measures during transients.

## 3. Properties of transient quality characteristics

In this section, two transient quality characteristics are investigated in Section 3.1. The second largest eigenvalue (SLE) of the state transition probability matrix characterizes the duration of system transients. And the pre-exponential coefficients (PEC) can be seen as the impact that the SLE has on the transients of product quality performance. The monotonic properties of SLE and PEC with respect to system parameters are explored by extensive numerical experiments in Section 3.2, and Section 3.3, respectively.

### 3.1. Transient quality characteristics

For two-stage manufacturing systems defined by assumptions 1–6, it follows from mathematical models that $A_2$ is the state transition probability matrix of an ergodic Markov chain, thus it has a unique largest eigenvalue equal to one. Arrange all the four eigenvalues of $A_2$ as follows:

$$1 = \lambda_1 > \lambda_2 \geqslant |\lambda_3| \geqslant |\lambda_4|$$

Based on matrix theory, there exists a non-singular matrix $Q$, with which $A_2$ can be transformed to the diagonal matrix whose diagonal elements are the eigenvalues of $A_2$. This is called matrix diagonalization. In mathematical form, we have

$$QA_2Q^{-1} = diag[1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4]$$

where $Q^{-1}$ is the inverse matrix of $Q$.

Introduce the following substitution

$$\widetilde{X}_2(t) = QX_2(t) \tag{8}$$

Substitute Eq. (8) into Eqs. (5)–(7), thus Eqs. (5)–(7) are transformed to

$$\widetilde{X}_2(t+1) = \widetilde{A}_2\widetilde{X}_2(t) \tag{9}$$

$$y_2(t) = \widetilde{C}\widetilde{X}_2(t) \tag{10}$$

where

$$\widetilde{A}_2 = QA_2Q^{-1} = diag\,[1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4]$$

$$\widetilde{C} = CQ^{-1} = \begin{bmatrix} \widetilde{C}_{11} & \widetilde{C}_{12} & \widetilde{C}_{13} & \widetilde{C}_{14} \\ \widetilde{C}_{21} & \widetilde{C}_{22} & \widetilde{C}_{23} & \widetilde{C}_{24} \end{bmatrix}$$

According to Eq. (9), the evolution of system states is denoted as

$$\widetilde{X}_2(t) = \widetilde{A}_2^t\widetilde{X}_2(0) = diag\,[1 \quad \lambda_2^t \quad \lambda_3^t \quad \lambda_4^t]\widetilde{X}_2(0) \tag{11}$$

where

$$\widetilde{X}_2(0) = QX_2(0)$$

Eqs. (8) and (11) show that Markov chain $\widetilde{X}_2(t)$ and $X_2(t)$ approach their steady states as exponential functions of time $t$ with parameter $\lambda_i$. Among the four eigenvalues, since the unique largest eigenvalue is equal to one, it's obvious that the second largest eigenvalue (SLE) of $A_2$ characterizes the duration of system transients.

As shown in Eq. (10), the evolution of $P(g_2,t)$ and $P(d_2,t)$ can be denoted as,

$$\begin{bmatrix} P(g_2,t) \\ P(d_2,t) \end{bmatrix} = \begin{bmatrix} \widetilde{C}_{11} & \widetilde{C}_{12} & \widetilde{C}_{13} & \widetilde{C}_{14} \\ \widetilde{C}_{21} & \widetilde{C}_{22} & \widetilde{C}_{23} & \widetilde{C}_{24} \end{bmatrix} diag\,[1 \quad \lambda_2^t \quad \lambda_3^t \quad \lambda_4^t]\widetilde{X}_2(0) \tag{12}$$

Considering that the first element of $\widetilde{X}_2(0)$ is $\widetilde{X}_{2,1}(0) = 1$ (the first row of $Q$ is the left eigenvector of $A_2$ given by $[1 \quad 1 \quad 1 \quad 1]$), it follows that,

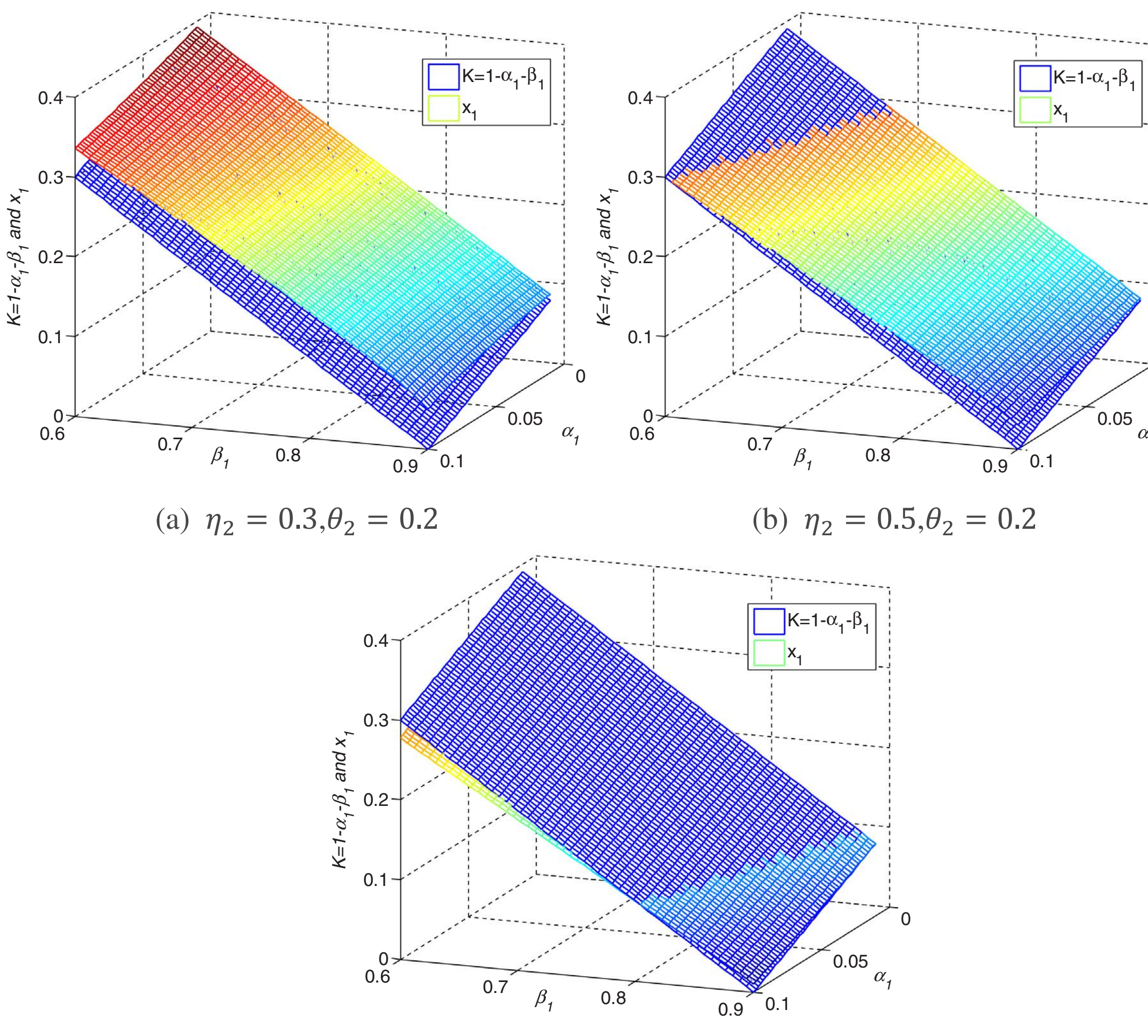
$$\begin{bmatrix} P(g_2,t) \\ P(d_2,t) \end{bmatrix} = \begin{bmatrix} \widetilde{C}_{11} + \widetilde{C}_{12}\widetilde{X}_{2,2}(0)\lambda_2^t + \widetilde{C}_{13}\widetilde{X}_{2,3}(0)\lambda_3^t + \widetilde{C}_{14}\widetilde{X}_{2,4}(0)\lambda_4^t \\ \widetilde{C}_{21} + \widetilde{C}_{22}\widetilde{X}_{2,2}(0)\lambda_2^t + \widetilde{C}_{23}\widetilde{X}_{2,3}(0)\lambda_3^t + \widetilde{C}_{24}\widetilde{X}_{2,4}(0)\lambda_4^t \end{bmatrix} \tag{13}$$

Denote $P(g_2)_{SS}$ and $P(d_2)_{SS}$ as the steady-state probability to produce good product of the system and the probability to produce defective product, respectively, and we have

$$P(g_2)_{SS} = \lim_{t\to\infty} P(g_2,t) = \widetilde{C}_{11} \tag{14}$$

$$P(d_2)_{SS} = \lim_{t\to\infty} P(d_2,t) = \widetilde{C}_{21} \tag{15}$$

It follows that




(a) $\eta_2 = 0.3, \theta_2 = 0.2$

(b) $\eta_2 = 0.5, \theta_2 = 0.2$

(c) $\eta_2 = 0.5, \theta_2 = 0.4$

**Fig. 4.** SLE as a function of $\alpha_1$ and $\beta_1$.

$$\begin{bmatrix} P(g_2,t) \\ P(d_2,t) \end{bmatrix} = \begin{bmatrix} P(g_2)_{SS}\left(1 + \frac{\widetilde{C}_{12}}{\widetilde{C}_{11}}\widetilde{X}_{2,2}(0)\lambda_2^t + \frac{\widetilde{C}_{13}}{\widetilde{C}_{11}}\widetilde{X}_{2,3}(0)\lambda_3^t + \frac{\widetilde{C}_{14}}{\widetilde{C}_{11}}\widetilde{X}_{2,4}(0)\lambda_4^t\right) \\ P(d_2)_{SS}\left(1 + \frac{\widetilde{C}_{22}}{\widetilde{C}_{21}}\widetilde{X}_{2,2}(0)\lambda_2^t + \frac{\widetilde{C}_{23}}{\widetilde{C}_{21}}\widetilde{X}_{2,3}(0)\lambda_3^t + \frac{\widetilde{C}_{24}}{\widetilde{C}_{21}}\widetilde{X}_{2,4}(0)\lambda_4^t\right) \end{bmatrix} \tag{16}$$

Eq. (16) suggests that the transients of $P(g_2,t)$ and $P(d_2,t)$ are characterized not only by the eigenvalues $\lambda_i$ of transition matrix $A_2$ but also by the pre-exponential coefficients (PECs), i.e., $\frac{\widetilde{C}_{ij}}{\widetilde{C}_{i1}}$, where $i = 1,2, j = 2,3,4$. Since $\lambda_2$ is the SLE, to be consistent, the most important PECs are $\frac{\widetilde{C}_{12}}{\widetilde{C}_{11}}$ and $\frac{\widetilde{C}_{22}}{\widetilde{C}_{21}}$. And we have

$$\Phi_1 = \left|\frac{\widetilde{C}_{12}}{\widetilde{C}_{11}}\right|, \Phi_2 = \left|\frac{\widetilde{C}_{22}}{\widetilde{C}_{21}}\right| \tag{17}$$

Coefficients $\Phi_1$ and $\Phi_2$ can be seen as the impact that the SLE has on the transients of product quality performance. The larger the coefficients, the larger the impact is.

As mentioned above, the SLE and PEC characterize the system quality transients. To investigate the properties of these transient quality characteristics, extensive numerical experiments have been carried out by randomly selecting the parameters of two-stage manufacturing systems defined by assumptions 1–6 in a reasonable set.

In our model, the value ranges of quality failure probabilities and quality repair probabilities generally are [0,1]. In the numerical experiments, considering actual production conditions, the value ranges are narrowed down according to some works (Wang, Li, Arinez, & Biller, 2013; Wang et al., 2010) and it is assumed that

(1) Quality failure probabilities with good coming parts are relatively small, $\alpha_1 \in [0,0.1]$ and $\gamma_2 \in [0,0.1]$.
(2) Quality repair probabilities with good coming parts are relatively large, $\beta_1 \in [0.6,0.9]$ and $\mu_2 \in [0.6,0.9]$.
(3) Quality failure probabilities and quality repair probabilities with defective coming parts, $\eta_2 \in [0,0.6]$ and $\theta_2 \in [0,0.4]$.

### 3.2. Analysis of the SLE

The SLE of the system states transition probability matrix $A_2$, i.e., $\lambda_2$, characterizes the duration of system transients. The rate of system convergence is described by SLE approximately. Larger SLE indicates slower convergence and longer duration of system transients.

In the framework of model defined by assumptions 1–6, the SLE is a function of the system parameters. Based on matrix theory, the characteristic polynomial of transition probability matrix $A_2$ is

$$|\lambda I - A_2| = (\lambda-1)(\lambda-K)[\lambda^2 - \lambda(M(1-\alpha_1) + N(1-\beta_1)) + MNK] \tag{18}$$

where $I$ is identity matrix, and

$$K = 1-\alpha_1-\beta_1, M = 1-\mu_2-\gamma_2, N = 1-\theta_2-\eta_2 \tag{19}$$

In the numerical analysis, for simplification, we consider the case where the transition probabilities with good coming parts are identical for $M_1$ and $M_2$, denoted as the equal stage case, i.e.,

$$\alpha_1 = \gamma_2, \beta_1 = \mu_2 \tag{20}$$

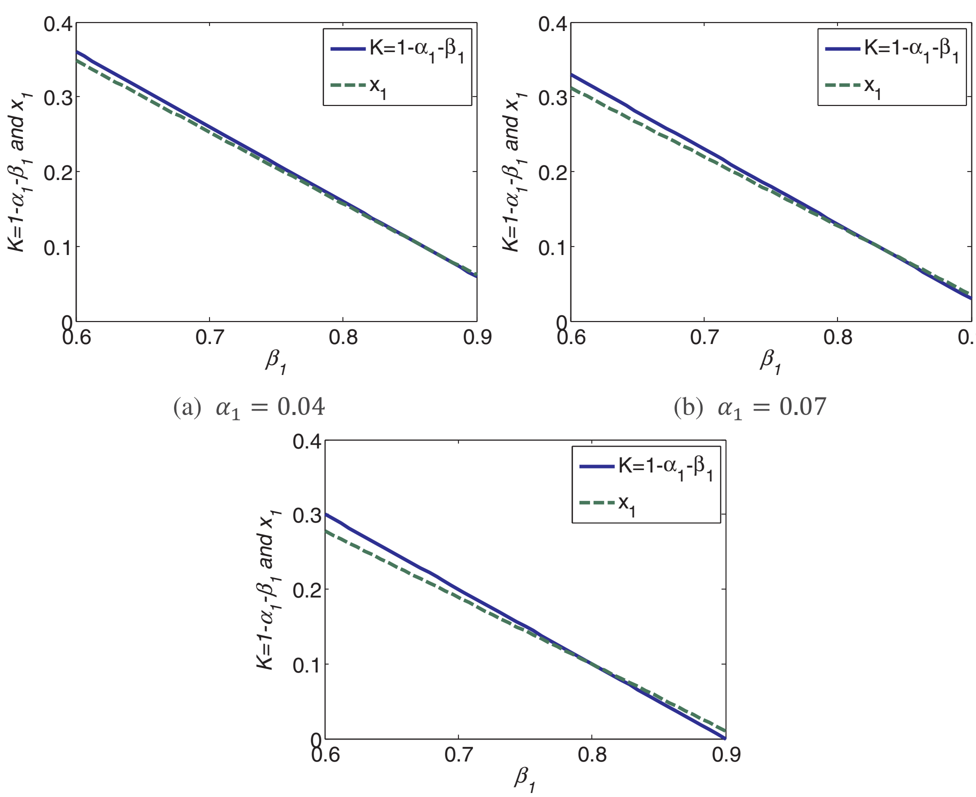
Then, Eq. (18) can be simplified as follows,

$$|\lambda I - A_2| = (\lambda-1)(\lambda-K)[\lambda^2 - \lambda(K(1-\alpha_1) + N(1-\beta_1)) + NK^2] \tag{21}$$

leading to the four eigenvalues of matrix $A_2$. Denoting $x_1$ and $x_2$ as follows,

$$x_1 = \frac{1}{2}\{(K(1-\alpha_1) + N(1-\beta_1)) + \sqrt{(K(1-\alpha_1) + N(1-\beta_1))^2 - 4NK^2}\} x_2$$
$$= \frac{1}{2}\{(K(1-\alpha_1) + N(1-\beta_1)) - \sqrt{(K(1-\alpha_1) + N(1-\beta_1))^2 - 4NK^2}\} \tag{22}$$

Thus, among the four eigenvalues $1, K, x_1, x_2$, the SLE can be either $K$ or $x_1$, depending on the value of $K$ and $x_1$. If $K > x_1$, the SLE of transition probability matrix $A_2$ is $K$; if the inequality is reversed, the SLE of




(a) $\alpha_1 = 0.04$

(b) $\alpha_1 = 0.07$

(c) $\alpha_1 = 0.1$

**Fig. 5.** SLE as a function of $\beta_1$, while $\eta_2 = 0.5$, $\theta_2 = 0.4$.

transition probability matrix $A_2$ is $x_1$.

In order to explore the properties of transient quality metric SLE as a function of system parameters $\alpha_1$, $\beta_1$, $\eta_2$ and $\theta_2$, extensive numerical analysis are carried out by selecting the system parameters randomly and equiprobably from the sets in Section 3.1. Firstly, the properties of SLE regarding $\alpha_1$ and $\beta_1$ are studied, and then the properties of SLE regarding $\eta_2$ and $\theta_2$ are studied.

With respect to $\alpha_1$ and $\beta_1$, due to space limitation, three typical examples are shown in Fig. 4 instead of all the manufacturing systems explored extensively. The three example groups of systems are with (a) $\eta_2 = 0.3$, $\theta_2 = 0.2$, (b) $\eta_2 = 0.5$, $\theta_2 = 0.2$, (c) $\eta_2 = 0.5$, $\theta_2 = 0.4$, respectively.

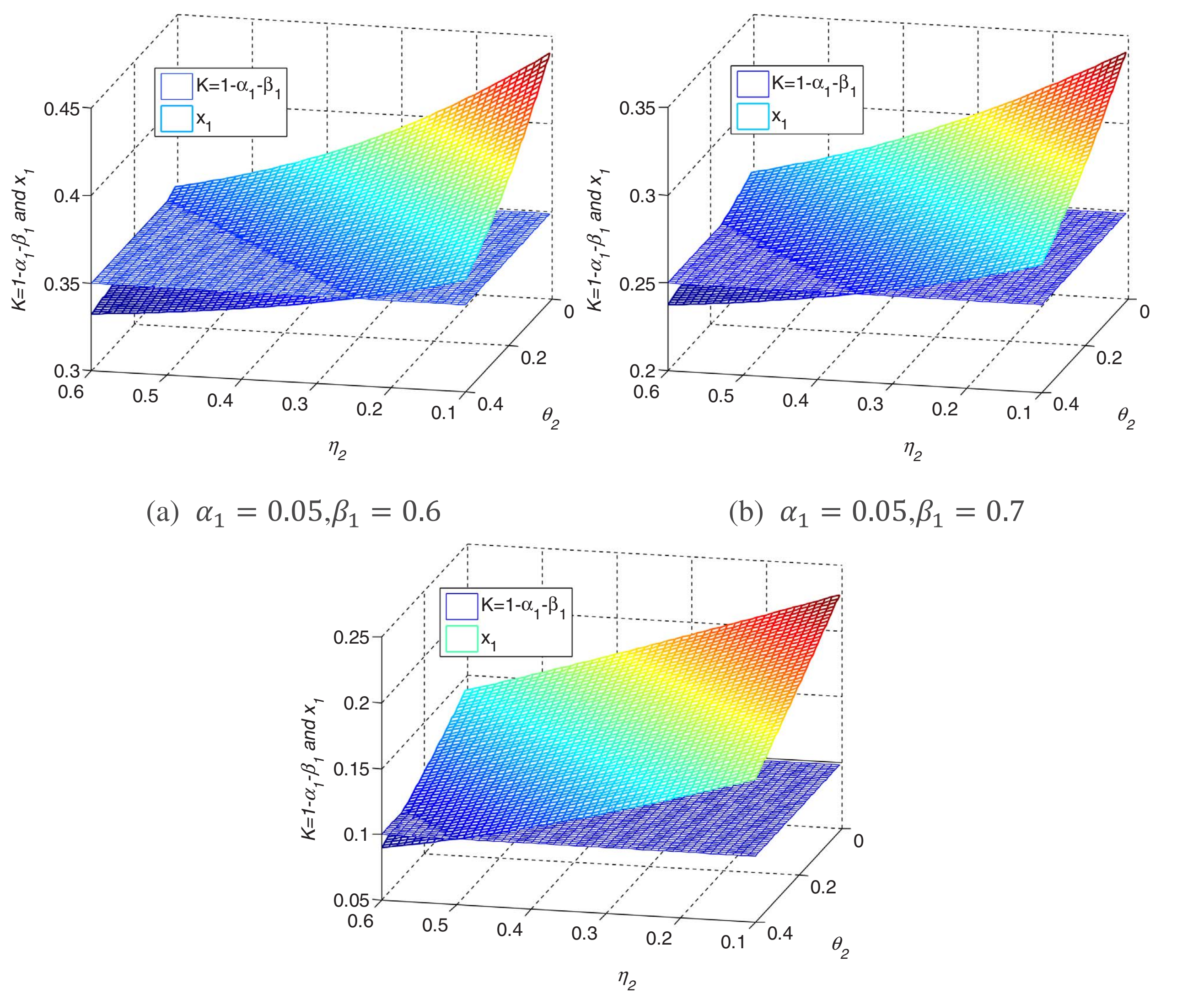
The monotonic properties of SLE regarding system parameters $\alpha_1$ and $\beta_1$ are plotted in Fig. 4 for the three example groups of systems. As shown in Fig. 4, for every group of system, there are two surfaces plotted, i.e., one surface is eigenvalue $K$ and the other is eigenvalue $x_1$. As mentioned previously, the larger value between $K$ and $x_1$ is the SLE. For small $\beta_1$, $K > x_1$ and $K$ is the SLE. As $\beta_1$ increases, $K$ and $x_1$ intersect. For large $\beta_1$, $K < x_1$ and $x_1$ is the SLE. Both $K$ and $x_1$ are monotonically decreasing in $\alpha_1$, and monotonically decreasing in $\beta_1$. Thus, SLE (the larger value between $K$ and $x_1$) is monotonically decreasing in $\alpha_1$ and $\beta_1$. More explicitly, the behavior of SLE with respect to $\beta_1$ is shown in 2D graph in Fig. 5. From Figs. 4 and 5, the following result is concluded.

*Numerical Result 1.* SLE is a monotonically decreasing function of $\alpha_1$ and $\beta_1$.

**Remark 1.** Note that the three example groups of systems in Fig. 4 are presented for demonstration. Actually, Numerical Result 1 is observed for two-stage manufacturing systems defined by assumptions 1–6 on a general basis and not only for the three groups of systems demonstrated. The same is true for Numerical Results 2 through 9.

Similarly, with respect to $\eta_2$ and $\theta_2$, three typical examples are shown in Fig. 6 instead of all the manufacturing systems explored. The three example groups of systems are with (a) $\alpha_1 = 0.05$, $\beta_1 = 0.6$, (b) $\alpha_1 = 0.05$, $\beta_1 = 0.7$, (c) $\alpha_1 = 0.1$, $\beta_1 = 0.8$, respectively.

The monotonic properties of SLE regarding system parameters $\eta_2$ and $\theta_2$ are plotted in Fig. 6 for the three example groups of systems. As shown in Fig. 6, for every group of system, there are two surfaces plotted, i.e., one surface is eigenvalue $K$ and the other is eigenvalue $x_1$. The larger value between $K$ and $x_1$ is the SLE. For small $\eta_2$ and $\theta_2$, $K < x_1$ and $x_1$ is the SLE. As $\eta_2$ and $\theta_2$ increase, $K$ and $x_1$ intersect. For large $\eta_2$ and $\theta_2$, $K > x_1$ and $K$ is the SLE. $K$ does not change with $\eta_2$ and $\theta_2$. $x_1$ is monotonically decreasing in $\eta_2$, and monotonically decreasing in $\theta_2$. Thus, for small $\eta_2$ and $\theta_2$, SLE (the larger value between $K$ and $x_1$, here is $x_1$) is monotonically decreasing in $\eta_2$ and $\theta_2$; as $\eta_2$ and $\theta_2$ increase, SLE (here is $K$) keeps a constant and does not change with $\eta_2$ and $\theta_2$, which




(a) $\alpha_1 = 0.05, \beta_1 = 0.6$

(b) $\alpha_1 = 0.05, \beta_1 = 0.7$

(c) $\alpha_1 = 0.1, \beta_1 = 0.8$

**Fig. 6.** SLE as a function of $\eta_2$ and $\theta_2$.

means SLE has a lower bound. More explicitly, the behavior of SLE with respect to $\theta_2$ is shown in 2D graph in Fig. 7. Clearly, for small $\eta_2$, SLE (here is $x_1$) is monotonically decreasing in $\theta_2$ (see Fig. 7(a)); however, for large $\eta_2$, SLE (here is $x_1$) firstly decreases in $\theta_2$ for small $\theta_2$ and then SLE (here is $K$) keeps a constant, thus SLE has a lower bound (see Fig. 7(c)). From Figs. 6 and 7, the following result is drawn.

*Numerical Result 2:* For small $\eta_2$ and $\theta_2$, SLE is a monotonically decreasing function of $\eta_2$ and $\theta_2$; as $\eta_2$ and $\theta_2$ increase, SLE keeps a constant and does not change with $\eta_2$ and $\theta_2$, i.e., SLE has a lower bound regarding $\eta_2$ and $\theta_2$.

**Remark 2.** From Numerical Results 1 and 2, when $\alpha_1$, $\beta_1$, $\eta_2$ or $\theta_2$ increases, the SLE decreases which leads to shorter duration of transients and faster convergence. However, unexpectedly, SLE shows a lower bound for larger $\eta_2$ and $\theta_2$. When $\eta_2$ and $\theta_2$ are increased to a certain extent, the SLE does not decline anymore. Thus it is a better choice to increase $\alpha_1$ and $\beta_1$ than $\eta_2$ and $\theta_2$ to reach faster transients.

### 3.3. Analysis of the PEC

As mentioned above, the transients of product quality performance are characterized by both the SLE and the PEC. The PECs are the coefficients pre-exponential of SLE. The PECs can be viewed as the impact to what extent the SLE has on system quality transients. Larger PECs indicate larger impact and slower transients.

Under model assumptions 1–6, the PEC is a function of the system parameters. To investigate the properties of PEC, similarly to analysis of SLE in Section 3.2, extensive numerical analysis is carried out by selecting the system parameters randomly and equiprobably. In the numerical analysis, for simplification, the equal stage case is still considered, i.e., when Eq. (20) holds. Based on Eqs. (10) and (17), firstly, the properties of PEC regarding $\alpha_1$ and $\beta_1$ are studied, and then the properties of PEC regarding $\eta_2$ and $\theta_2$ are studied.

With respect to system parameters $\alpha_1$ and $\beta_1$, the monotonic properties of PEC are plotted in Fig. 8(a)–(c) for the three typical example groups of systems. As shown in Fig. 8(a)–(c), PEC is monotonically increasing in $\alpha_1$. Generally, PEC is monotonically decreasing in $\beta_1$. When $\theta_2$ is quite large ($\theta_2 = 0.4$), PEC demonstrates a slight increase (less than 0.005) in $\beta_1$. More explicitly, the behavior of PEC with respect to $\beta_1$ is shown in 2D graph in Fig. 8(d)–(e). PEC is generally decreasing in $\beta_1$ (see Fig. 8(d)) and slightly increasing in $\beta_1$ (see Fig. 8(e)). From Fig. 8, the following result is concluded.

*Numerical Result 3:* PEC is a monotonically increasing function of $\alpha_1$. It is a monotonically decreasing function of $\beta_1$.

Similarly, the monotonic properties of PEC regarding system parameters $\eta_2$ and $\theta_2$ are plotted in Fig. 9(a)–(c) for the three example groups of systems. As shown in Fig. 9(a)–(c), PEC is monotonically decreasing in $\eta_2$, and monotonically decreasing in $\theta_2$. More explicitly, the behavior of PEC with respect to $\theta_2$ is shown in 2D graph in Fig. 9(d). From Fig. 9, the following result is drawn.

*Numerical Result 4:* PEC is a monotonically decreasing function of $\eta_2$ and $\theta_2$.

**Remark 3.** From Numerical Result 3 and 4, as $\beta_1$, $\eta_2$ and $\theta_2$ increase, or as $\alpha_1$ decreases, the PEC decreases. The effects of SLE on the evolution of system states diminish and is favorable for a shorter duration of transients.

## 4. Settling time

In terms of throughput analysis for production systems, Zhang et al. (2013) has introduced the concept of settling time to describe the time needed for production rate and work-in-process to reach the steady state. Similar to throughput analysis, settling time is introduced to

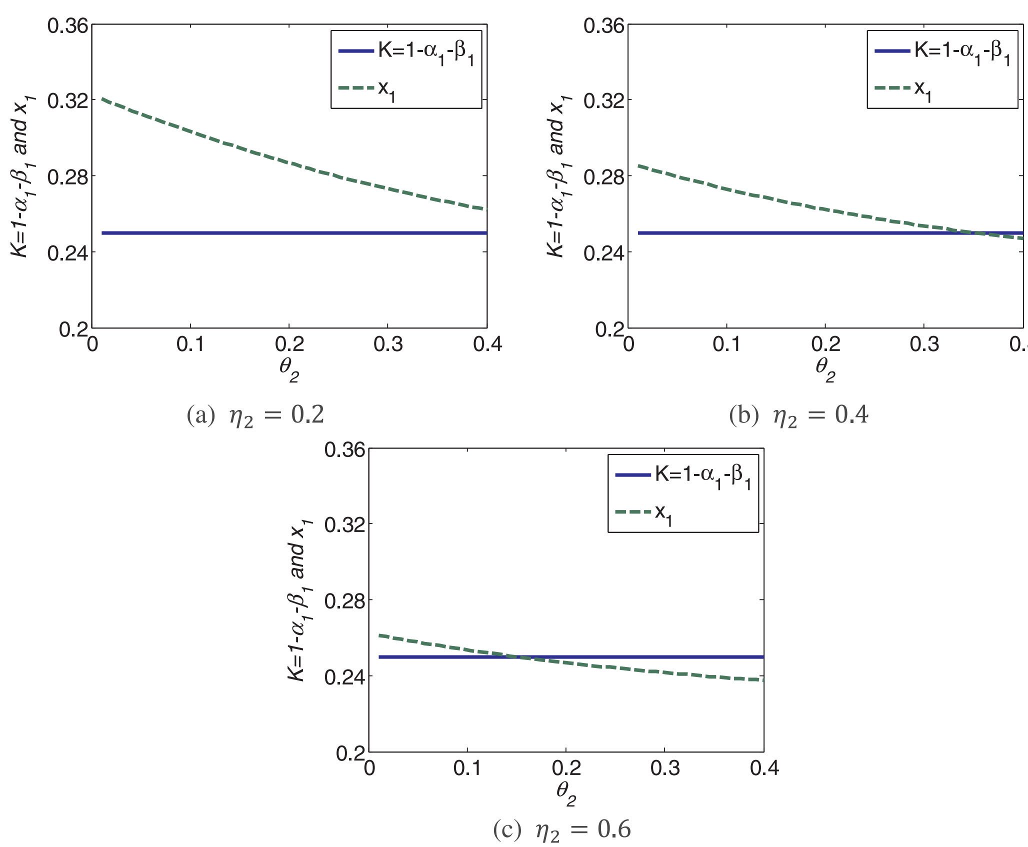
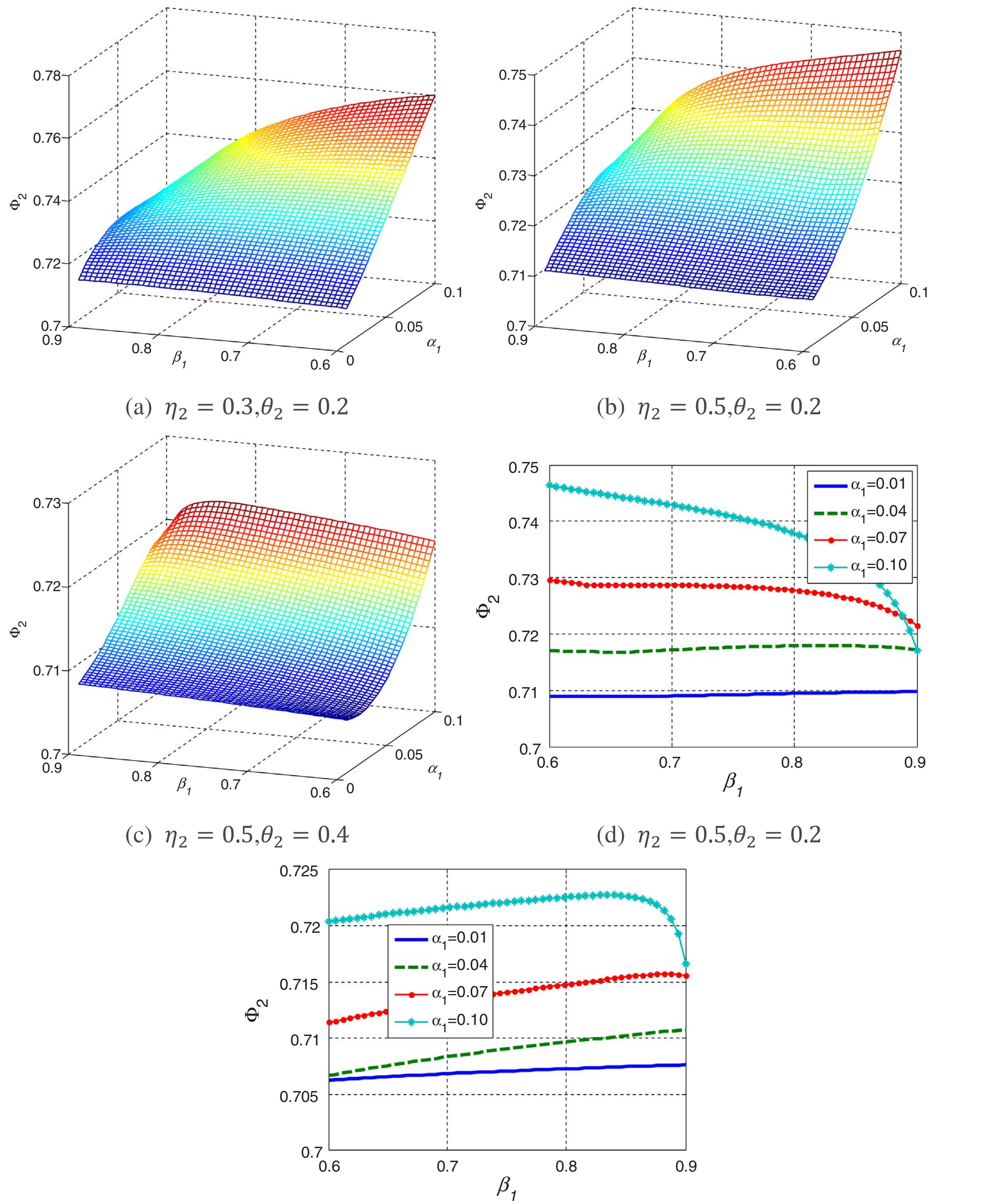



**Fig. 7.** SLE as a function of $\theta_2$, while $\alpha_1 = 0.05$, $\beta_1 = 0.7$.




(a) $\eta_2 = 0.3, \theta_2 = 0.2$

(b) $\eta_2 = 0.5, \theta_2 = 0.2$

(c) $\eta_2 = 0.5, \theta_2 = 0.4$

(d) $\eta_2 = 0.5, \theta_2 = 0.2$

(e) $\eta_2 = 0.5, \theta_2 = 0.4$

**Fig. 8.** PEC as a function of $\alpha_1$ and $\beta_1$.

describe the duration of system transients in transient quality analysis. The settling time is denoted as the time needed for the system quality performance $P(g_2,t)$ to reach and remain within $\pm$ 3% of its steady state value. We define the settling time of $P(g_2,t)$ as follows:

$$t_S = \inf\left( t \;\middle|\; \frac{P(g_2)_{SS} - P(g_2,t)}{P(g_2)_{SS}} \leqslant 3\% \right) \tag{23}$$

Thus before investigation of $t_S$, we first analyze the evolution of $P(g_2,t)$ as a function of $t$.

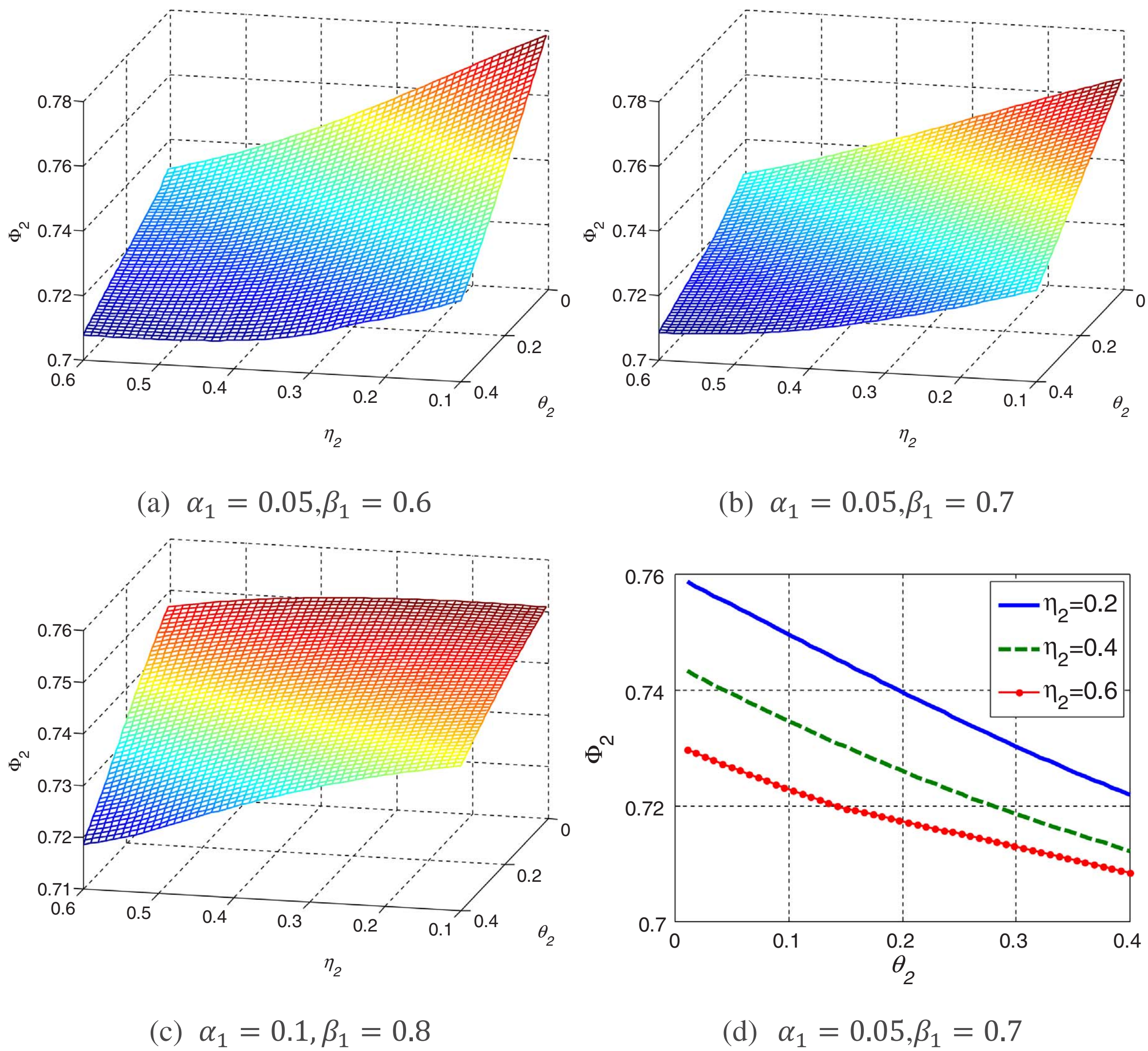



**Fig. 9.** PEC as a function of $\eta_2$ and $\theta_2$.

### 4.1. Behavior of $P(g_2,t)$

According to Eq. (13), the exact evolution of $P(g_2,t)$ can be obtained. In this section, an approximation of $P(g_2,t)$ can be developed based on the SLE:

$$\widehat{P(g_2,t)} = P(g_2)_{SS}[1-\omega\lambda_2^t] \tag{24}$$

where $P(g_2)_{SS}$ is solved by Eq. (14) and $\lambda_2$ is the SLE. To solve the coefficient $\omega$, consider the approximation equality $P(g_2,1) = \widehat{P(g_2,1)}$, where $P(g_2,1)$ is solved by Eq. (13). Thus Eq. (24) can be used to approximate $P(g_2,t)$.

The accuracy investigation of approximation (24) is shown in Fig. 10. The exact calculation of $P(g_2,t)$ solved by Eq. (13) is also plotted in Fig. 10 for comparison. From Fig. 10, the approximation (24) can track the real transient quality performance closely. More explicitly, the accuracy of approximation is quantitatively defined as,

$$\delta_{P(g_2)} = \max_{t=1,2,\cdots} \frac{|P(g_2,t)-\widehat{P(g_2,t)}|}{P(g_2)_{SS}} \times 100\% \tag{25}$$

Eq. (24) approximates the evolution of $P(g_2,t)$ with respect to $t$, and it can be also used for the approximation of settling time. Following the definition of settling time (23) and Eq. (24), we have

$$P(g_2)_{SS}[1-\omega\lambda_2^{\widehat{t_S}}] = \widehat{P(g_2,\widehat{t_S})} \geqslant (1-3\%)P(g_2)_{SS}$$

The solution $\widehat{t_S}$ is the approximation of settling time,

$$\widehat{t_S} = \frac{\ln[3/(100\omega)]}{\ln\lambda_2} \tag{26}$$

The estimate accuracy is quantitatively defined as,

$$\delta_{t_S} = |t_S-\widehat{t_S}| \tag{27}$$

To evaluate the effectiveness of settling time approximation (26), extensive numerical experiments are carried out by selecting the system parameters randomly and equiprobably from the parameter sets in Section 3.1, with $t_S$ solved by exact calculation and $\widehat{t_S}$ solved by approximation. The cumulative frequency for $\delta_{t_S}$ is illustrated in Fig. 11 according to Eq. (27). In more than 95% of all cases examined, the estimate $\widehat{t_S}$ is within one time slot from the real value $t_S$, which validates the effectiveness of the approximation.

### 4.2. Analysis of settling time

Based on Eqs. (14) and (16), extensive numerical analysis is carried out to investigate the properties of settling time $t_S$ as a function of the system parameters. In the numerical analysis, for simplification, the equal stage case is still considered, i.e., when Eq. (20) holds. Firstly, the properties of $t_S$ regarding $\alpha_1$ and $\beta_1$ are studied, and then the properties of $t_S$ regarding $\eta_2$ and $\theta_2$ are studied.

With respect to $\alpha_1$ and $\beta_1$, the monotonic properties of $t_S$ are plotted

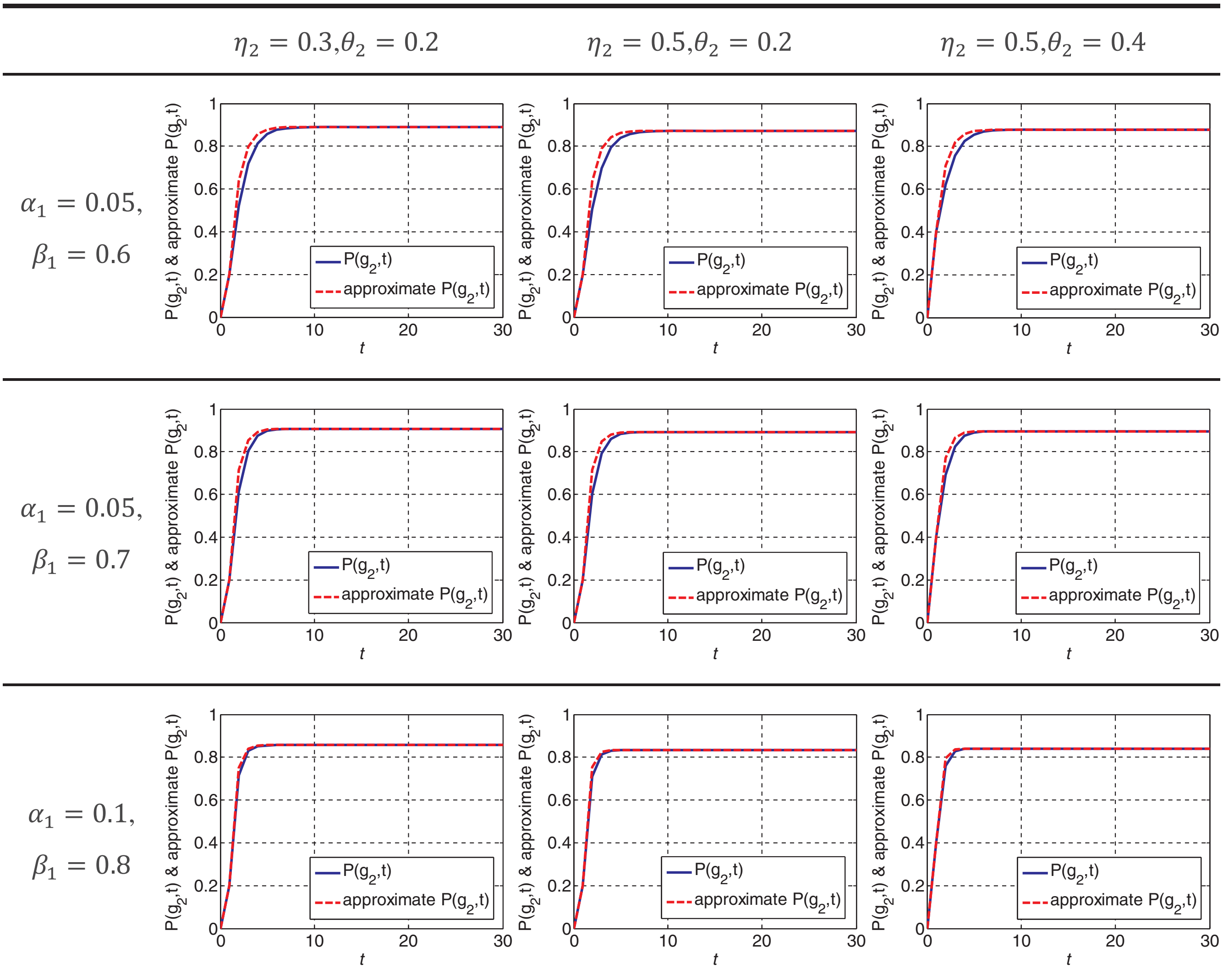



**Fig. 10.** The dynamics of $P(g_2,t)$ and its approximation $P(\hat{g}_2,t)$.

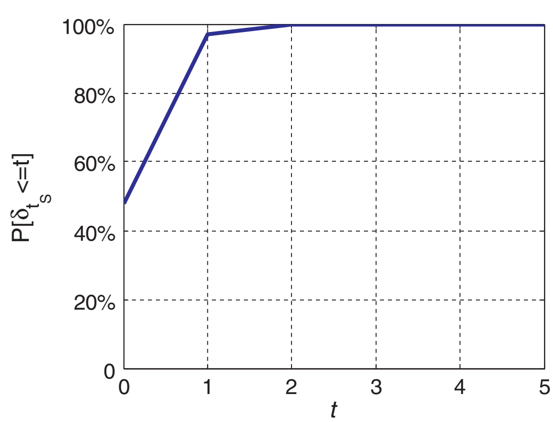



**Fig. 11.** Accuracy of approximation $\hat{t_S}$: the cumulative frequency for $\delta_{t_S}$.

in Fig. 12(a)–(c) for the three example groups of systems. As shown in Fig. 12(a)–(c), $t_S$ is decreasing in $\alpha_1$, and decreasing in $\beta_1$. More explicitly, the behavior of $t_S$ with respect to $\beta_1$ is shown in 2D graph in Fig. 12(d). From Fig. 12, the following result is concluded.

*Numerical Result 5:* Settling time $t_S$ is a decreasing function of $\alpha_1$ and $\beta_1$.

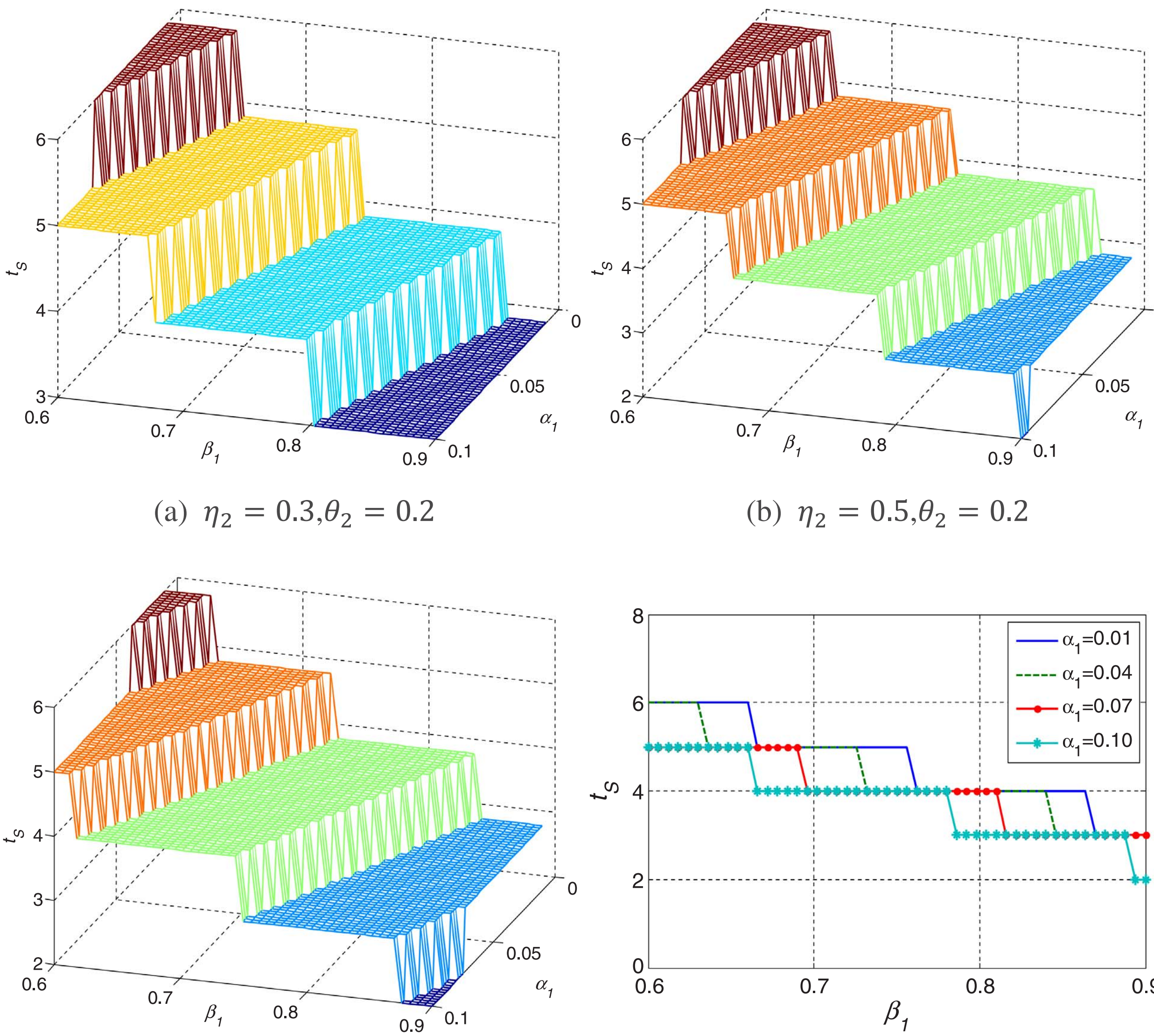
Similarly, the monotonic properties of $t_S$ regarding system parameters $\eta_2$ and $\theta_2$ are plotted in Fig. 13(a)–(c) for the three example groups of systems. As shown in Fig. 13(a)–(c), settling time is decreasing in $\eta_2$, and decreasing in $\theta_2$. More explicitly, the behavior of settling time with respect to $\theta_2$ is shown in 2D graph in Fig. 13(d). From Fig. 13, the following result is drawn.

*Numerical Result 6:* Settling time $t_S$ is a decreasing function of $\eta_2$ and $\theta_2$.

**Remark 4.** From Numerical Results 5 and 6, as $\alpha_1$, $\beta_1$, $\eta_2$ or $\theta_2$ increases, the settling time is generally reduced and the system undergoes a shorter transient. As a direct measure for transient duration, settling time is a combination effect of transient quality characteristics on system transients, i.e., the SLE and PEC. Thus, Numerical Results 5 and 6 are in accordance with Numerical Results 1–4 in general. Moreover, settling time is more sensitive to the changes of $\alpha_1$ and $\beta_1$ than $\eta_2$ and $\theta_2$. This indicates that improving $\alpha_1$ and $\beta_1$ can bring larger reduction in settling time on factory floor.

## 5. Quality loss

In this section, the system quality loss issue due to transients is analyzed in Section 5.1. The initial condition of the manufacturing system has strong impact on system quality transients. The monotonic




(a) $\eta_2 = 0.3, \theta_2 = 0.2$

(b) $\eta_2 = 0.5, \theta_2 = 0.2$

(c) $\eta_2 = 0.5, \theta_2 = 0.4$

(d) $\eta_2 = 0.5, \theta_2 = 0.2$

**Fig. 12.** Settling time as a function of $\alpha_1$ and $\beta_1$.

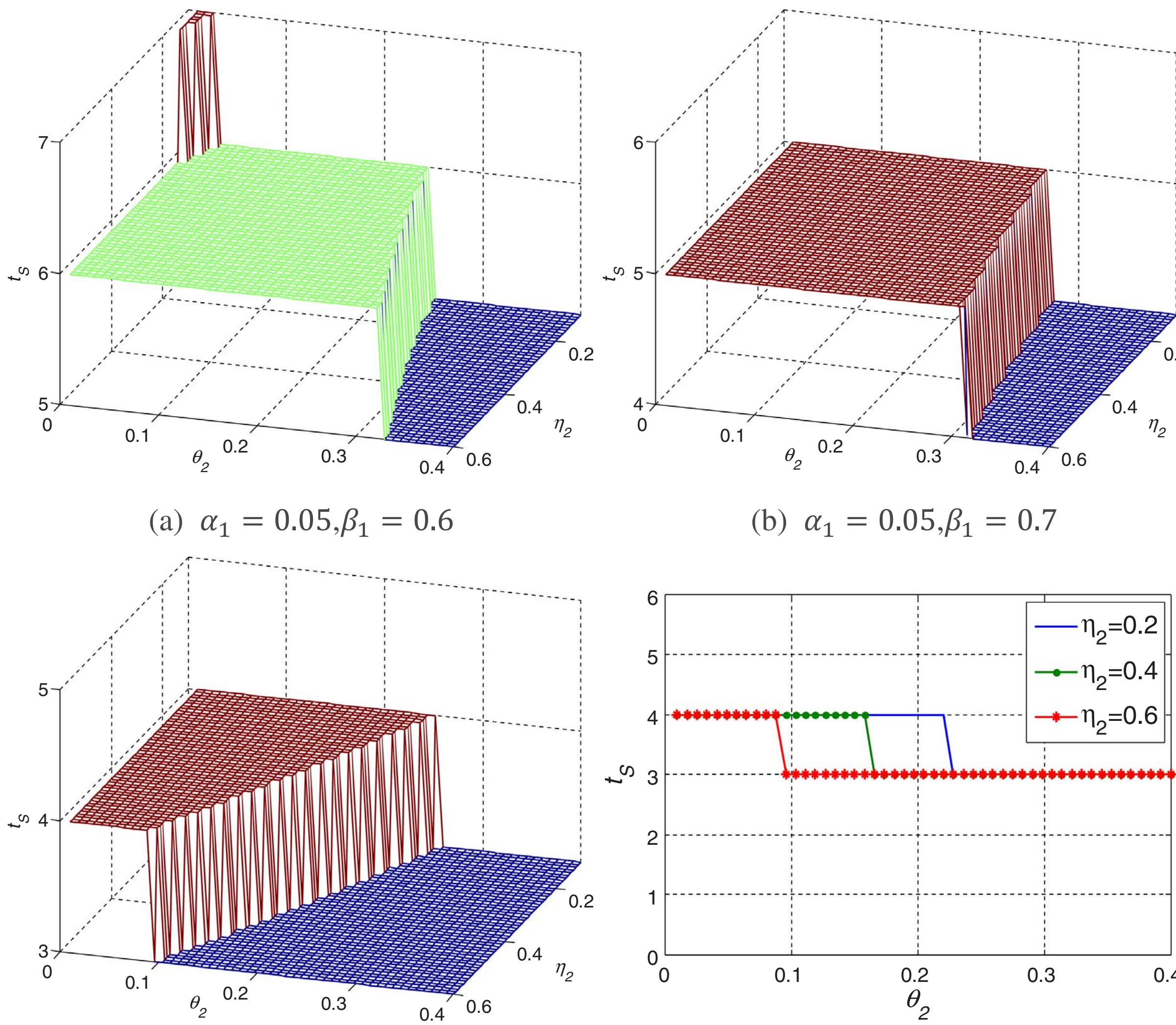
properties of quality loss during transients are explored in Section 5.2. However, system quality performance of steady state is quite different from that of transients. Thus it is important to fully analyze system quality in terms of both transients and steady state. The monotonic properties of steady state quality are explored to facilitate continuous improvement in Section 5.3. A detailed comparison is discussed between the method proposed in this paper and the state-of-the-art models in related literature recently in Section 5.4.

### 5.1. Quality loss due to transients

As analyzed previously, manufacturing systems suffer quality loss due to system transients. In fact, the initial condition of machines has strong impact on the transients of the system states and the quality performance measures. For a two-stage manufacturing system defined by assumptions 1–6, the system has four quality states at a certain time $t$ according to Section 2.2. During transients after a fresh restart of manufacturing systems caused by product changeover, machine maintenance, etc., manufacturing system is typically in the defective quality state dominated by fixture relocation errors. In other words, the initial state during transients is $d_1d_2$ which means that both $M_1$ and $M_2$ are producing defective products. As it follows from Eq. (1), the states of the Markov chain at time 0 is $X_2(0) = [0 \;\; 0 \;\; 0 \;\; 1]^T$. For system parameters $\alpha_1 = 0.1$, $\beta_1 = 0.7$, $\gamma_2 = 0.1$, $\mu_2 = 0.7$, $\eta_2 = 0.5$ and $\theta_2 = 0.2$, the dynamics of four system quality states and quality performance measure $P(g_2,t)$ are plotted in Fig. 14(a)–(b).

In contrast, consider an ideal initial case, i.e., the initial state is $g_1g_2$ which means that both $M_1$ and $M_2$ are producing good products. In this case, the states of Markov chain at time 0 is $X_2(0) = [1 \;\; 0 \;\; 0 \;\; 0]^T$. Again for system parameters $\alpha_1 = 0.1$, $\beta_1 = 0.7$, $\gamma_2 = 0.1$, $\mu_2 = 0.7$, $\eta_2 = 0.5$ and $\theta_2 = 0.2$, the dynamics of four system quality states and quality performance measure $P(g_2,t)$ are plotted in Fig. 15(a) and (b).

From Figs. 14 and 15, the system quality performance converges to the steady state in different manners, depending on the initial state of the manufacturing system. In the ideal case, system quality performance approaches its steady state from above the steady state value, which results in system quality gain (see Fig. 15(b)). However, actually, system quality performance approaches its steady state from below the steady state value, which results in system quality loss due to transients (see Fig. 14(b)).




(a) $\alpha_1 = 0.05, \beta_1 = 0.6$ (b) $\alpha_1 = 0.05, \beta_1 = 0.7$

(c) $\alpha_1 = 0.1, \beta_1 = 0.8$ (d) $\alpha_1 = 0.1, \beta_1 = 0.8$

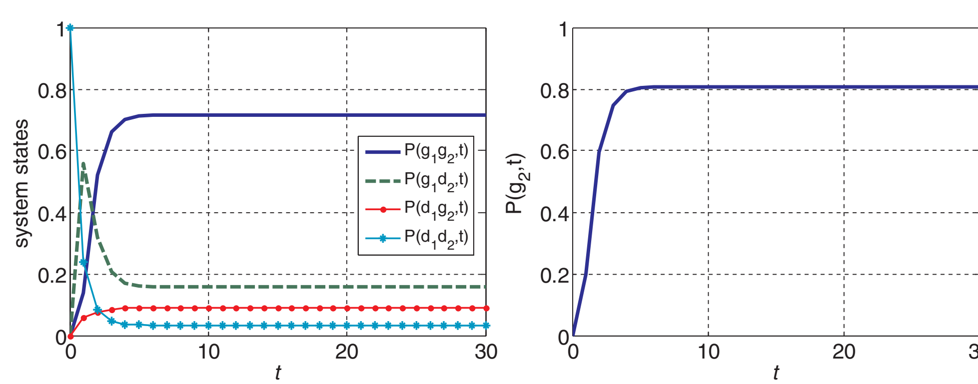
**Fig. 13.** Settling time as a function of $\eta_2$ and $\theta_2$.

### 5.2. Analysis of quality loss during transients

To formulate the quality loss due to transients and analyze the structural properties of quality loss regarding system parameters, denote quality loss for a period $T$ as:

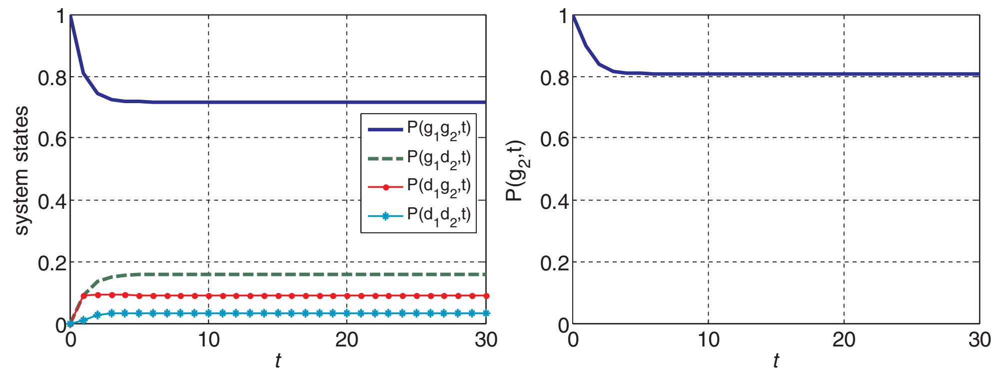
$$L_Q(X_2(0)) = \sum_{t=0}^{T} [P(g_2)_{SS} - P(g_2,t;X_2(0))] \tag{28}$$

And the percent of quality loss compared to steady state is defined as:




(a) system states (b) quality performance measure

**Fig. 14.** Transients of system states and quality performance with the actual initial condition.

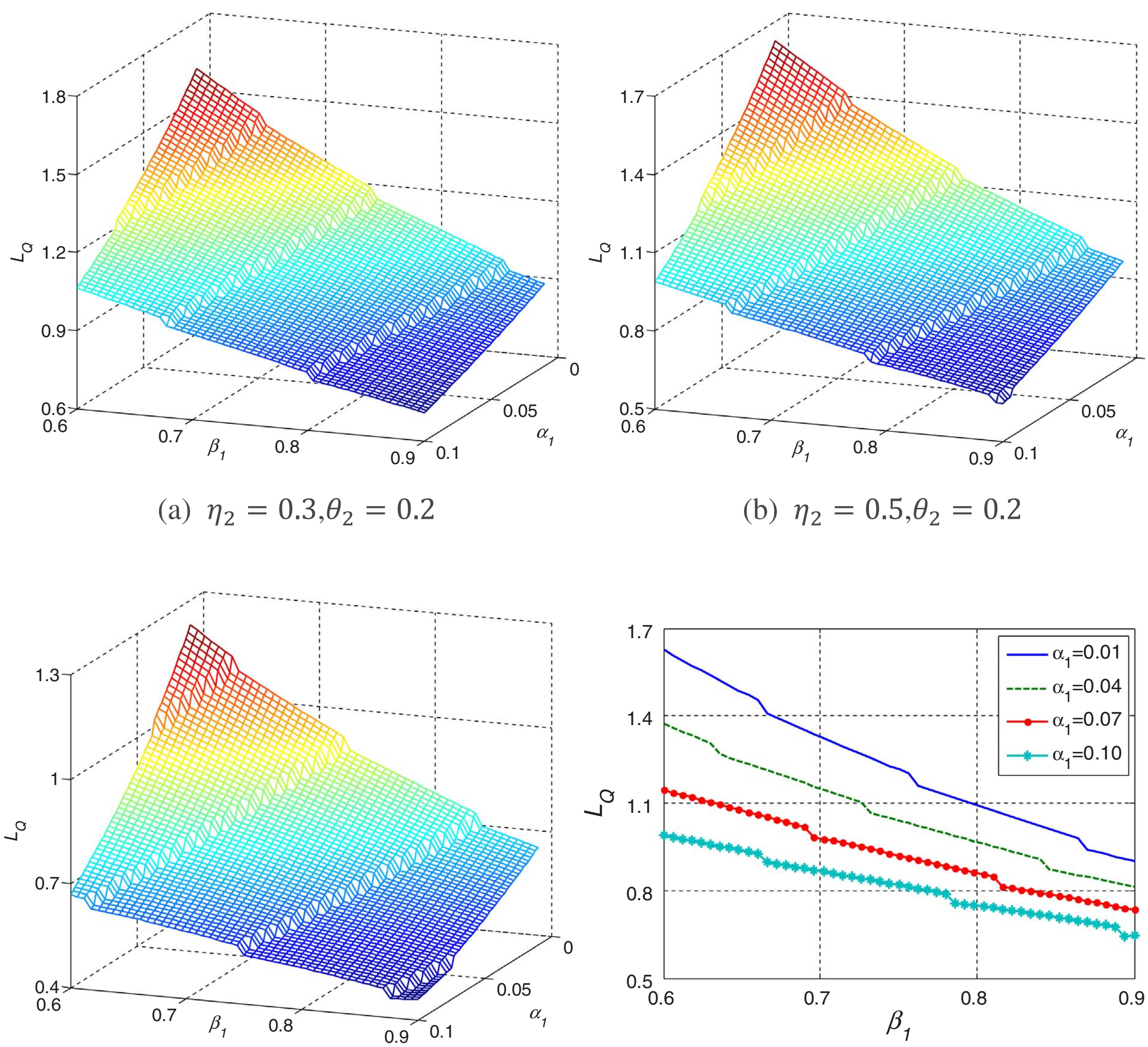



(a) system states　(b) quality performance measure

**Fig. 15.** Transients of system states and quality performance with the ideal initial case.

$$\Omega_Q = \frac{L_Q(X_2(0))}{T \times P(g_2)_{SS}} \times 100\% \tag{29}$$

Extensive numerical analysis has been carried out to investigate the properties of quality loss issue. In the numerical analysis, for simplification, the equal stage case is still considered, i.e., when Eq. (20) holds. Based on Eqs. (28) and (29), firstly, the properties of quality loss regarding $\alpha_1$ and $\beta_1$ are explored, and then the properties of quality loss




(a) $\eta_2 = 0.3, \theta_2 = 0.2$　(b) $\eta_2 = 0.5, \theta_2 = 0.2$

(c) $\eta_2 = 0.5, \theta_2 = 0.4$　(d) $\eta_2 = 0.5, \theta_2 = 0.2$

**Fig. 16.** Quality loss as a function of $\alpha_1$ and $\beta_1$.

regarding $\eta_2$ and $\theta_2$ are explored.

With respect to $\alpha_1$ and $\beta_1$, the monotonic properties of quality loss during transients are plotted in Fig. 16(a)–(c) for the three example groups of systems. As shown in Fig. 16(a)–(c), quality loss is monotonically decreasing in $\alpha_1$, and monotonically decreasing in $\beta_1$. More explicitly, the behavior of quality loss with respect to $\beta_1$ is shown in 2D graph in Fig. 16(d). From Fig. 16, the following result is concluded.

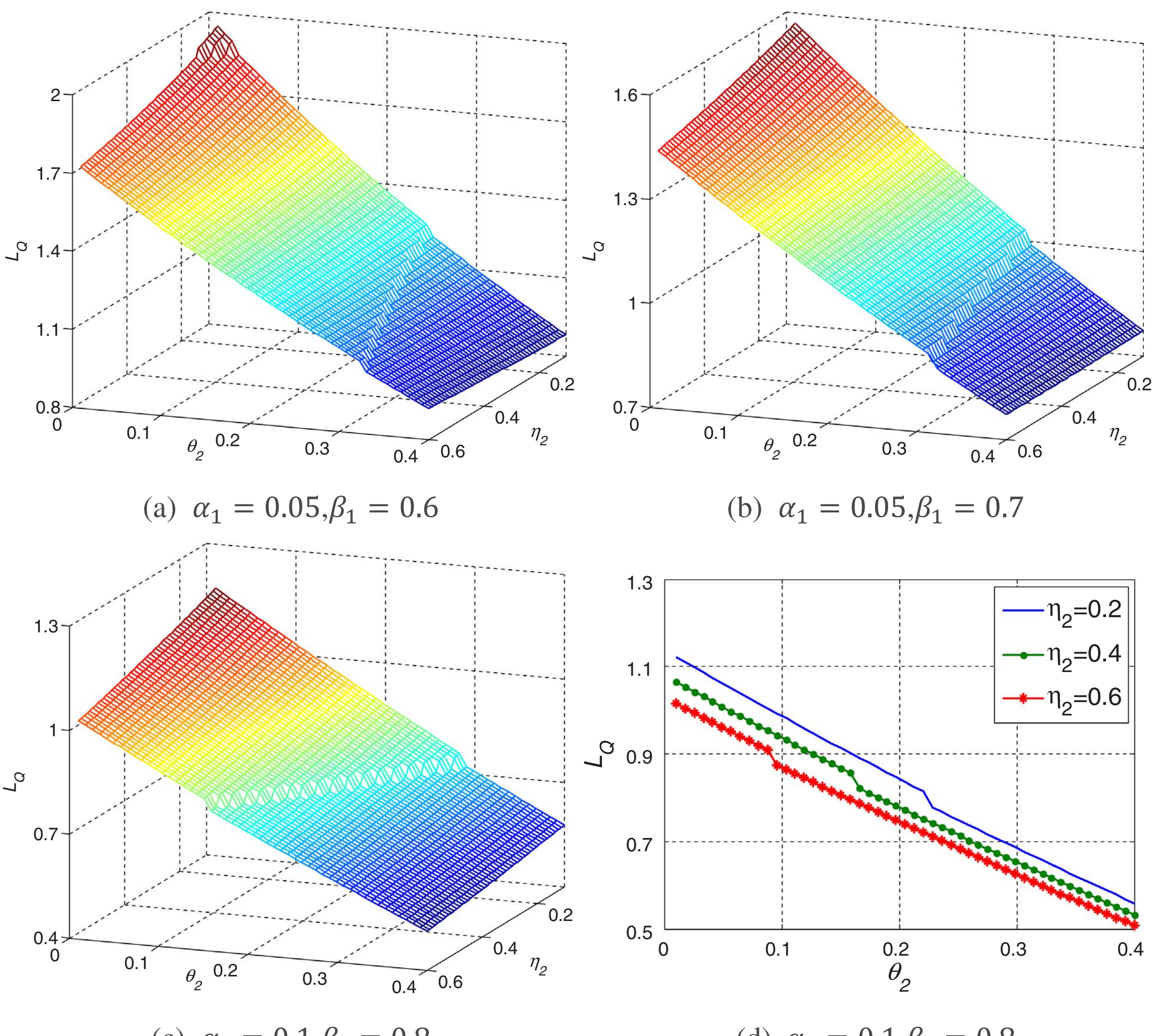
*Numerical Result 7:* Quality loss during transients is a monotonically decreasing function of $\alpha_1$ and $\beta_1$.

**Remark 5.** Note that an interesting phenomenon is observed from Fig. 12(d) and 16(d). The settling time in Fig. 12(d) and quality loss in Fig. 16(d) are both piecewise functions of $\beta_1$. In addition, the turning point of piecewise functions are almost the same for each corresponding curve in two figures. Before the turning point, the settling time is stable for a range of $\beta_1$, while quality loss shows a gentle decrease as $\beta_1$ increases. At the turning point, the settling time decreases, while quality loss shows a steep decline and the same point of piecewise is presented. From above analysis, it appears that a strong correlation exists between quality loss $L_Q$ and settling time $t_S$. This phenomenon can be seen as a proof that longer duration of transients causes greater quality loss generally. The same is true for settling time and quality loss regarding system parameters with defective coming parts (see Fig. 13(d) and 17(d)).

Similarly, the monotonic properties of quality loss during transients regarding system parameters $\eta_2$ and $\theta_2$ are plotted in Fig. 17(a)–(c) for the three example groups of systems. As it follows from Fig. 17(a)–(c), quality loss is monotonically decreasing in $\eta_2$, and monotonically decreasing in $\theta_2$. More explicitly, the behavior of quality loss with respect to $\theta_2$ is shown in 2D graph in Fig. 17(d). From Fig. 17, the following result is drawn.

*Numerical Result 8:* Quality loss during transients is a monotonically decreasing function of $\eta_2$ and $\theta_2$.

**Remark 6.** As it follows from Numerical Results 7 and 8, increasing $\alpha_1$, $\beta_1$, $\eta_2$ or $\theta_2$ practically leads to reduction in quality loss during transients. Considering the sensitivity of the settling time (see Remark 4) and the correlation between settling time and quality loss (see Remark 5), it suggests that increasing $\alpha_1$ and $\beta_1$ is more favorable than $\eta_2$ and $\theta_2$.




(a) $\alpha_1 = 0.05, \beta_1 = 0.6$ (b) $\alpha_1 = 0.05, \beta_1 = 0.7$

(c) $\alpha_1 = 0.1, \beta_1 = 0.8$ (d) $\alpha_1 = 0.1, \beta_1 = 0.8$

**Fig. 17.** Quality loss as a function of $\eta_2$ and $\theta_2$.

### 5.3. Continuous improvement

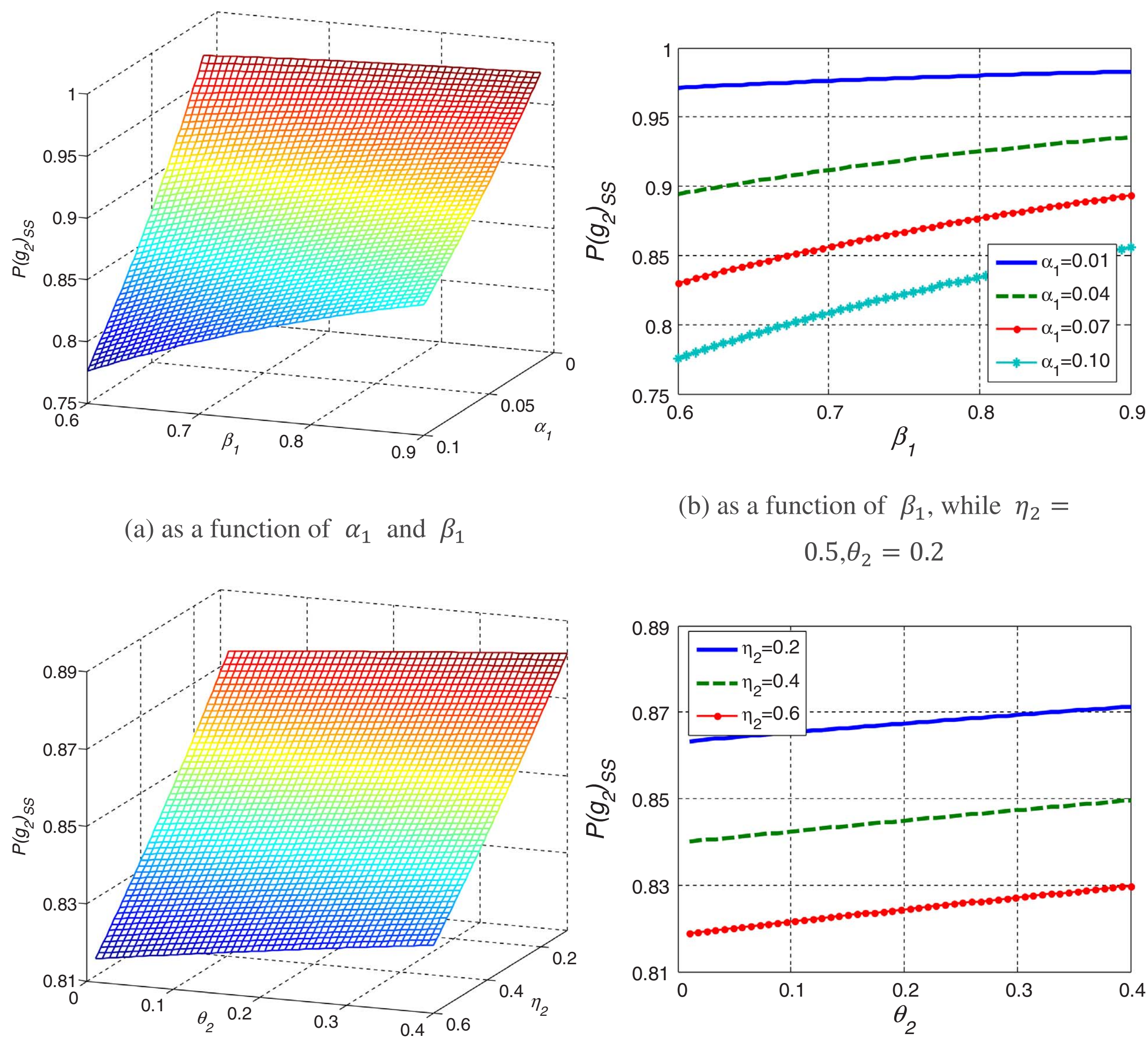
It is of critical significance to improve quality and reduce cost in flexible manufacturing systems. According to analysis of quality loss during transients, Remark 6 provides us guidance for planning continuous improvement. However, the results of the monotonic properties of system quality performance in Section 5.2 is based on the system transients, and does not consider the steady state. In this section it is shown that such monotonic properties may not hold. In other words, quality performance of steady state is quite different from that of transients. Thus there is still need to fully analyze the quality performance especially in terms of both transients and steady state.

As it follows from Eq. (14), extensive numerical analysis is carried out to investigate the monotonic properties of steady state quality performance by selecting the system parameters randomly and equiprobably. In the numerical analysis, the equal stage case is still considered, i.e., when Eq. (20) holds. As an illusion, the monotonic properties of steady state quality regarding system parameters $\alpha_1$ and $\beta_1$ are plotted in Fig. 18 (a)-(b), and the monotonic properties of steady state quality regarding system parameters $\eta_2$ and $\theta_2$ are presented in Fig. 18(c)-(d).

In Fig. 18(a)-(b), steady state quality performance is monotonically decreasing in $\alpha_1$, and monotonically increasing in $\beta_1$. In Fig. 18(c)-(d), steady state quality performance is monotonically decreasing in $\eta_2$, and monotonically increasing in $\theta_2$. From Fig. 18, the following result is drawn.

*Numerical Result 9:* Steady state quality performance is a monotonically decreasing function of $\alpha_1$ and $\eta_2$. It is a monotonically increasing function of $\beta_1$ and $\theta_2$.

**Remark 7.** The qualitative effect of quality failure probabilities $\alpha_1$ and $\eta_2$ on the transients of system quality performance differs from that on




(a) as a function of $\alpha_1$ and $\beta_1$

(b) as a function of $\beta_1$, while $\eta_2 = 0.5, \theta_2 = 0.2$

(c) as a function of $\eta_2$ and $\theta_2$

(d) as a function of $\theta_2$, while $\alpha_1 = 0.1, \beta_1 = 0.8$

**Fig. 18.** Steady state quality as a function of system parameters.

steady state quality. Although increasing $\alpha_1$ and $\eta_2$ will lead to reduction of quality loss due to transients, it is detrimental to quality performance in steady state. Fortunately, increasing $\beta_1$ and $\theta_2$ leads to both quality loss reduction during transients and quality improvement in steady state. Thus it is desired to increase $\beta_1$ and $\theta_2$ than increase $\alpha_1$ and $\eta_2$ in order to reduce quality loss and facilitate continuous improvement, which provides practical directions for operation management in flexible systems.

### *5.4. Comparison with the state-of-the-art*

In this subsection we make a detailed comparison between the method proposed in this paper and the state-of-the-art models in related literature. Recent papers for comparison are Ju, Li, Xiao, Arinez, and Deng (2015), Ju et al. (2016, 2017), Lee, Li, Musa, Bain, and Nelson (2017), Zhong, Lee, and Li (2017) and Jia and Zhang (2017).

Firstly, comparisons are made in the modeling of quality propagation in manufacturing systems between this paper and Ju et al. (2015, 2016). In fact, the two Markov models focus on different kinds of manufacturing systems. In multi-stage manufacturing systems, the variations of the final product quality are the accumulation of variations introduced and propagated as workpieces move through all the stages. In Ju et al. (2015, 2016), the assembly system has inspection stations and repair stations after each stage (denoted as ubiquitous inspection systems). In this paper, the manufacturing systems with RQIF have only one inspection station at the last stage of the manufacturing system and no repair stations exist. There exist significant differences in the way of quality propagation between these two kinds of systems. For systems in Ju et al. (2015, 2016), since every stage has an inspection station after the stage, only good quality products are passed on to downstream stage. Thus the coming parts for every stage are all with good quality, and the quality corrections by the system itself are not considered. But for systems with RQIF, since the product defects are only inspected and identified at the end of the production line, a defective product from upstream stages will not go out of the system until the last stage and they may be corrected by downstream stages. In other words, the quality of the product manufactured by stage $M_i (i \geqslant 2)$ relies on both the state of stage $M_i$ (there exist not only quality degradation but also quality correction by the system), and the quality of the coming part from upstream stage $M_{i-1}$ (the coming parts for each stage may be good or defective before being processed).

Moreover, the key objectives of the two models are different although they're both based on state transition probabilities and Markov methods. In Ju et al. (2015, 2016), the aim is to analyze system quality performance in steady-state phase. While in this paper, the Markov modeling is intended to conduct analysis of quality performance during transients. Lee et al. (2017) and Zhong et al. (2017) consider Markov modeling of patient transitions and medication error propagation in health care systems. Similarly, the coming medication from upstream stage is with good state and quality corrections are not considered. Also, their models deal with steady-state analysis.

Secondly, comparisons are made in the context of transient analysis

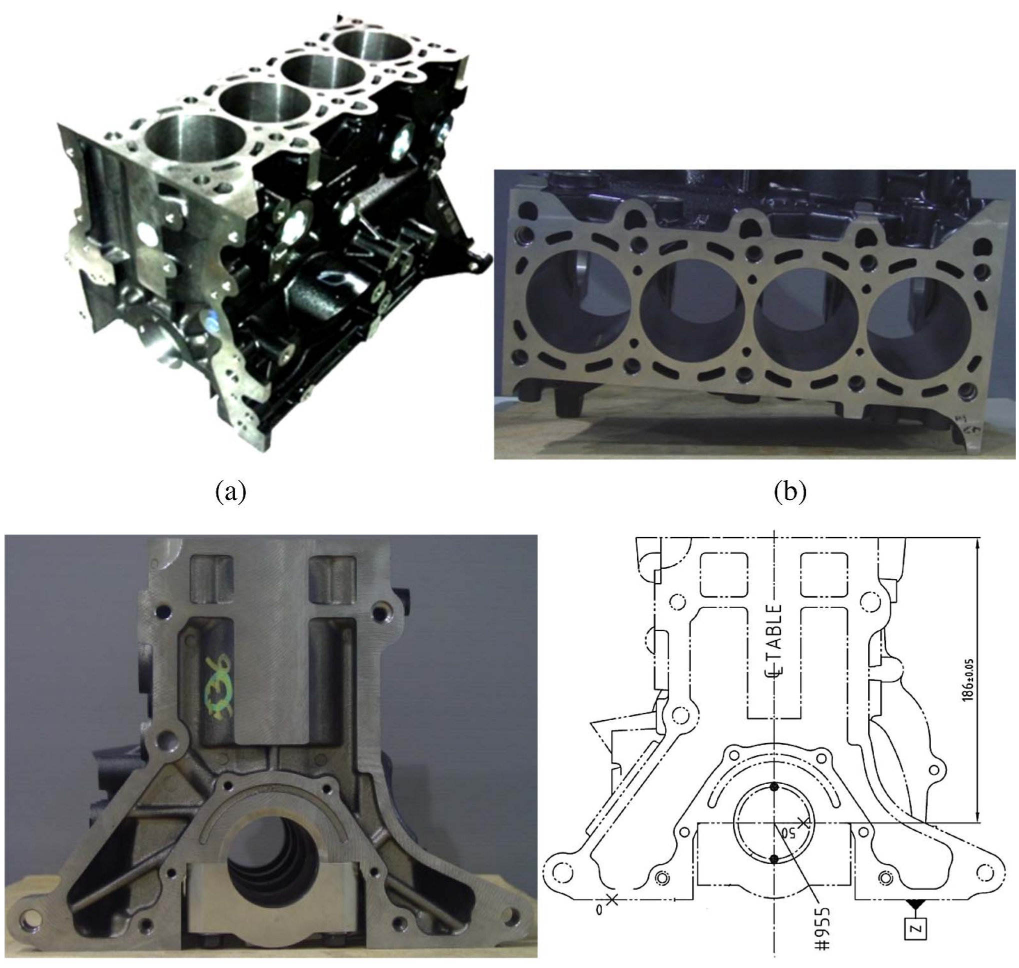



**Fig. 19.** Cylinder block and the product quality characteristic.

of manufacturing systems. Jia and Zhang (2017) and Ju et al. (2017) focus on the throughput analysis of production lines in the framework of bernoulli machines and finite buffers during transients. In this paper, a Markov model is developed to address the coupling between manufacturing system parameters and quality performance in terms of system transient duration. Specifically, an analytical method is proposed to deal with quality propagation in a two-stage manufacturing system with RQIF during transients. Transient quality performance metrics, including the real-time product quality, settling time, and quality loss due to transients, are derived. Although these papers are all explored in transient analysis recently, they focus on different aspects of performance evaluation in manufacturing systems.

In a word, the Markov method proposed in this paper is quite different from the related papers recently, as it is targeted at quality propagation in the specific manufacturing systems with RQIF during transients. Both the quality of coming parts and the states of stages are considered in such systems. In other words, the coming parts may be good or defective for each stage before being processed and there exist not only quality degradation but also quality correction in quality propagation.

## 6. Case study

In this section, a case study has been carried out at the flexible manufacturing line of engine cylinder block to validate the effectiveness of the proposed method. To ensure the confidentiality of the data, all the parameters utilized in case study have been modified. However, the nature of these data and system structural properties still hold.

### 6.1. Experimental setup

The manufacturing process of engine cylinder block is a typically complicated multiple stage process (more than twenty stages) with many key product characteristics. Here, we focus on the product quality characteristic of the distance between the cylinder block top face and the crankshaft hole. The three-dimensional profile of cylinder block, the top face and the profile are shown in Fig. 19(a)–(d). The corresponding manufacturing process is composed of two stages, i.e., OP 30 and OP 190 (OP represents the operation sequence numbers). OP 30 is the operation of semi milling the top face, and OP 190 is the operation of finish milling the top face. Product quality propagation is analyzed in this two-stage manufacturing system, i.e., the quality of the top face which is first machined in OP30 can be corrected or deteriorated in OP190.

The transition probabilities data are estimated based on historical processing data analysis. By implementing the steps in section 2.1, we get the transition probabilities data necessary in this case study.

### 6.2. Results and analysis

The transition probabilities of the two stages system are presented in the form of quality failure probabilities and quality repair probabilities in Fig. 20. All the probabilities are based on historical processing data on the factory floor.

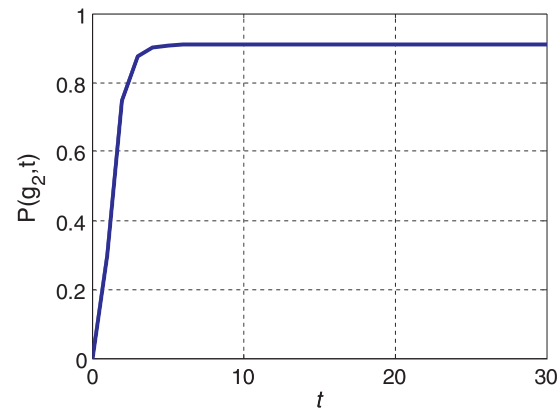



**Fig. 21.** The evolution of system quality performance during transients.

As shown in Fig. 20, $\alpha_1 = 0.05$, $\beta_1 = 0.8$, $\gamma_2 = 0.05$, $\mu_2 = 0.8$, $\eta_2 = 0.5$, $\theta_2 = 0.3$. Using these probabilities and the method of transient quality analysis proposed above, the steady state quality, settling time and quality loss due to transients can be calculated. The steady state quality is 90.95%. The settling time is 4 time slots, and the quality loss is 0.89%. The evolution of system quality performance $P(g_2,t)$ during transients is plotted in Fig. 21. These results are in accordance with the actual data measured on the factory floor, which demonstrate the effectiveness of the proposed method.

Next the monotonic property analysis and parameter sensitivity analysis are conducted to explore in what manner the changes of parameters affecting system transient quality and which one brings the largest quality improvement. The values of $\alpha_1$, $\beta_1$, $\eta_2$, $\theta_2$ are increased or decreased by given percentages. Specifically, they are modified by ± 10%, ± 15%, ± 20%, respectively.

The settling time, the quality loss, the steady state quality corresponding to these parameter changes are illustrated in Fig. 22(a)–(c). And the following results can be drawn:

(1) In Fig. 22(a), the monotonicity of settling time holds, which is consistent with Numerical Results 5–6 and Remark 4. The settling time will be shorten as $\beta_1$ increases. In this special case, since settling time is insensitive to the changes of other parameters in the given value ranges of transition probabilities above, it does not change as other parameters vary.
(2) In Fig. 22(b), the monotonicity of quality loss validates the effectiveness of Numerical Results 7–8 and Remark 6. The quality loss is a decreasing function of system parameters. And since the quality loss is most sensitive to $\beta_1$, it is better to improve $\beta_1$.
(3) In Fig. 22(c), the monotonicity of steady state quality proves Numerical Result 9 and Remark 7. The steady state quality is improved as $\beta_1$ or $\theta_2$ increase, and as $\alpha_1$ or $\eta_2$ decrease. Also $\beta_1$ is the most sensitive parameter.

In summary, the transient quality analysis of this two-stage manufacturing system provides the guidance for quality improvement. It is

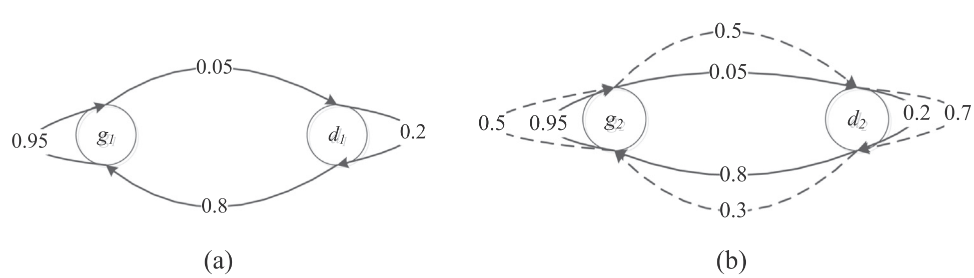



**Fig. 20.** State transition diagrams of the stages in the case.

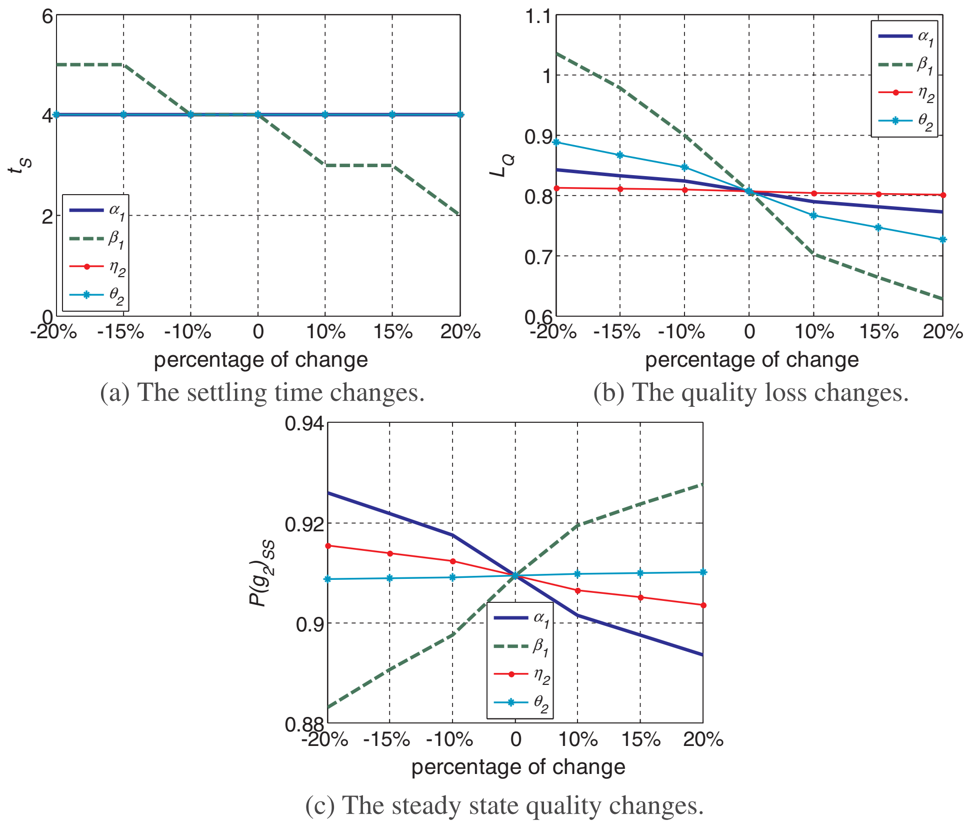



**Fig. 22.** Quality performance changes corresponding to changes of parameters.

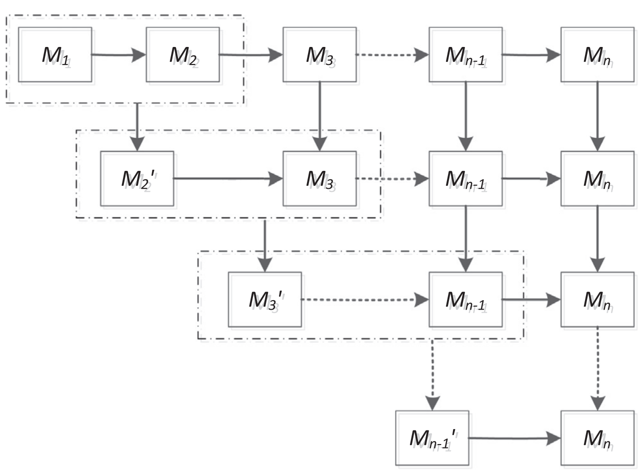



**Fig. 23.** Iterative procedures for multi-stage manufacturing systems.

better to increase $\beta_1$ to shorten the duration of transients, reduce quality loss due to transients, and improve steady state quality performance.

## 7. Conclusions and future work

This paper addresses the problem of transient analysis of quality performance in manufacturing systems. Specifically, an analytical method is developed using Markov model to address quality propagation in a two-stage manufacturing system with RQIF during transients. Based on the proposed mathematical model, analytical formulas for evaluating transient quality performance including the real-time product quality, settling time, and quality loss due to transients, are derived. In addition, the monotonicity properties of critical transient system characteristics and quality performance evaluation metrics are thoroughly explored. Extensive numerical experiments indicate that system quality transients are dominated by the SLE of the state transition matrix and that quality loss is tightly correlated with settling time. The behavior of quality performance during transients is quite different from that of steady state. It is desired to improve the quality repair probability with good coming parts in order to shorten duration of transients, reduce quality cost, and facilitate continuous improvement of quality performance in both transients and steady state. Finally, the proposed method is validated with case study on the factory floor, and the results demonstrate the effectiveness for transient quality analysis in two-stage manufacturing systems.

To extend the study, future research can be extended in three aspects.

(1) The generalization of the methods and results proposed in this paper to transient analysis of system quality performance in non-equal stage case.
(2) It is also possible to conduct the extension of the proposed approach to serial multi-stage manufacturing systems and more complex systems. For multi-stage manufacturing systems, the general iterative procedures are depicted in Fig. 23. First we derive the quality of the two-stage system $M_1$–$M_2$ by applying the Markov model derived in Section 2.2, and then merge stage $M_1$ and $M_2$ to one merged stage $M_2'$; then we model the quality of the new two-stage system $M_2'$–$M_3$, and then merge stage $M_2'$ and $M_3$ to one merged stage $M_3'$; continue the iterative process until the first $(n-1)$ stages are merged to one merged stage $M_{n-1}'$, and then model the quality of the final two-stage system $M_{n-1}'$–$M_n$. By implementing the iteration of a series of two-stage systems, the final quality of multi-stage manufacturing system is derived.
(3) Further research may also be devoted to expanding the results for transient modeling and analysis of multiple types of products.

## References


Abellan-Nebot, J. V., Liu, J., Subirón, F. R., & Shi, J. (2012). State space modeling of variation propagation in multi station machining processes considering machining-induced variations. *Journal of Manufacturing Science and Engineering, 134*(2), 90–98.

Chen, G., Wang, C., Zhang, L., Arinez, J., & Xiao, G. (2016). Transient performance analysis of serial production lines with geometric machines. *IEEE Transactions on Automatic Control, 61*(4), 877–891.

Chen, G., Zhang, L., Arinez, J., & Biller, S. (2013). Energy-efficient production systems through schedule-based operations. *IEEE Transactions on Automation Science and Engineering, 10*(1), 27–37.

Du, S., & Lv, J. (2013). Minimal Euclidean distance chart based on support vector regression for monitoring mean shifts of auto-correlated processes. *International Journal of Production Economics, 141*, 377–387.

Du, S., Lv, J., & Xi, L. (2012). On-line classifying process mean shifts in multivariate control charts based on multiclass support vector machines. *International Journal of Production Research, 50*, 6288–6310.

Du, S., & Xi, L. (2012). A robust approach for root causes identification in machining processes using hybrid learning algorithm and engineering knowledge. *Journal of Intelligent Manufacturing, 23*, 1833–1847.

Du, S., Xu, R., Huang, D., & Yao, X. (2015a). Markov modeling and analysis of multi-stage manufacturing systems with remote quality information feedback. *Computers & Industrial Engineering, 88*, 13–25.

Du, S., Yao, X., & Huang, D. (2015c). Engineering model-based Bayesian monitoring of ramp-up phase of multistage manufacturing process. *International Journal of Production Research, 53*(15), 4594–4613.

Du, S., Yao, X., Huang, D., & Wang, M. (2015b). Three-dimensional variation propagation modeling for multistage turning process of rotary workpieces. *Computers & Industrial Engineering, 82*, 41–53.

Huang, W., Lin, J., Bezdecny, M., Kong, Z., & Ceglarek, D. (2007a). Stream-of-variation modeling I: A generic 3D variation model for rigid body assembly in single station assembly processes. *ASME Transactions on Journal of Manufacturing Science and Engineering, 129*(4), 821–831.

Huang, W., Lin, J., Kong, Z., & Ceglarek, D. (2007b). Stream-of-variation modeling II: A generic 3D variation model for rigid body assembly in multi station assembly processes. *ASME Transactions on Journal of Manufacturing Science and Engineering, 129*(4), 832–842.

Inman, R. R., Blumenfeld, D. E., Huang, N., & Li, J. (2013). Survey of recent advances on the interface between production system design and quality. *IIE Transaction, 45*(6), 557–574.

Jia, Z., & Zhang, L. (2017). Transient performance analysis of closed production lines with bernoulli machines, finite buffers, and carriers. *IEEE Robotics & Automation Letters, 2*(4), 1893–1900.

Jia, Z., Zhang, L., Arinez, J., & Xiao, G. (2014). Analysis and improvement of batch-based production lines with Bernoulli machines. *IEEE International Conference on Automation Science and Engineering (CASE), 2014*, 13–18.

Jia, Z., Zhang, L., Arinez, J., & Xiao, G. (2015a). Transient performance evaluation of assembly systems with Bernoulli machines. *IFAC-Papers Online, 48*(3), 88–93.

Jia, Z., Zhang, L., Arinez, J., & Xiao, G. (2015b). Performance analysis of Bernoulli serial production lines with switch-on/off machine control. *IEEE International Conference on Automation Science and Engineering,* 477–482.

Jia, Z., Zhang, L., Arinez, J., & Xiao, G. (2016a). Finite production run-based serial lines with bernoulli machines: Performance analysis, bottleneck, and case study. *IEEE Transactions on Automation Science & Engineering, 13*(1), 134–148.

Jia, Z., Zhang, L., Chen, G., Arinez, J., & Xiao, G. (2016b). Performance evaluation in finite production run-based serial lines with geometric machines. *IEEE International Conference on Automation Science and Engineering,* 450–455.

Jin, J., & Shi, J. (1999). State space modeling of sheet metal assembly for dimensional control. *ASME Transactions on Journal of Manufacturing Science and Engineering, 121*(4), 756–762.

Ju, F., Li, J., & Horst, J. A. (2017). Transient analysis of serial production lines with perishable products: Bernoulli reliability model. *IEEE Transactions on Automatic Control, 62*(2), 694–707.

Ju, F., Li, J., Xiao, G., & Arinez, J. (2013). Quality flow model in automotive paint shops. *International Journal of Production Research, 51*(21), 6470–6483.

Ju, F., Li, J., Xiao, G., Arinez, J., & Deng, W. (2015). Modeling, analysis, and improvement of integrated productivity and quality system in battery manufacturing. *IIE Transactions, 47*(12), 1313–1328.

Ju, F., Li, J., Xiao, G., Huang, N., Arinez, J., Biller, S., & Deng, W. (2016). Improving battery manufacturing through quality and productivity bottleneck indicators. *Advances in Battery Manufacturing, Service, and Management Systems,* 29–56.

Ju, F., Li, J., Xiao, G., Huang, N., & Biller, S. (2014). A quality flow model in battery manufacturing systems for electric vehicles. *IEEE Transaction on Automation Science and Manufacturing, 11*(1), 230–244.

Kim, J., & Gershwin, S. B. (2008). Analysis of long flow lines with quality and operational failure. *IIE Transactions, 40*(3), 284–296.

Kim, W. S., & Morrison, J. R. (2015). The throughput rate of serial production lines with deterministic process times and random setups: Markovian models and applications to semiconductor manufacturing. *Computers & Operations Research, 53*, 288–300.

Lee, H. K., Li, J., Musa, A. J., Bain, P. A., & Nelson, K. (2017). Modeling and analysis of patient transitions in community hospitals: A systems approach. IEEE Transactions on Systems, Man, and Cybernetics: Systems (in press). http://dx.doi.org/10.1109/TSMC.2017.2723559.

Montgomery, D. C. (2009). *Introduction to statistical quality control* (6th ed.). NJ: John Wiley & Sons Inc.

Ossai, C. I., Boswell, B., & Davies, I. J. (2016). A Markovian approach for modelling the effects of maintenance on downtime and failure risk of wind turbine components. *Renewable Energy, 96*, 775–783.

Shao, X., Zhong, X., Li, J., Gewertz, B. L., Catchpole, K., Ley, E. J., & Wiegmann, D. A. (2015). Bottleneck analysis to reduce surgical flow disruptions: Theory and application. *IEEE Transactions on Automation Science and Engineering, 12*(1), 127–139.

Shi, J. (2006). *Stream of variation modeling and analysis for multistage manufacturing processes.* New York: CRC Press.

Shi, J., & Zhou, S. (2009). Quality control and improvement for multistage systems: A survey. *IIE Transactions, 41*(9), 744–753.

Tang, D., Yu, J., Chen, X., & Makis, V. (2015). An optimal condition-based maintenance policy for a degrading system subject to the competing risks of soft and hard failure. *Computers & Industrial Engineering, 83*, 100–110.

Wang, J., Hu, Y., & Li, J. (2010). Transient analysis to design buffer capacity in dairy filling and packing production lines. *Journal of Food Engineering, 98*(1), 1–12.

Wang, Y., & Li, L. (2015). A novel modeling method for both steady-state and transient analyses of serial Bernoulli production systems. *IEEE Transactions on Systems, Man, and Cybernetics: Systems, 45*(1), 97–108.

Wang, J., Li, J., Arinez, J., & Biller, S. (2012). Indicators for quality improvability and bottleneck sequence in flexible manufacturing systems with batch productions. *International Journal of Production Research, 50*, 6388–6402.

Wang, J., Li, J., Arinez, J., & Biller, S. (2013). Quality bottleneck transitions in flexible manufacturing systems with batch productions. *IIE Transactions, 45*, 190–205.

Wang, J., Zhong, X., Li, J., & Howard, P. K. (2014). Modeling and analysis of care delivery services within patient rooms: A system-theoretic approach. *IEEE Transactions on Automation Science and Engineering, 11*(2), 379–393.

Wu, C. H., Chien, W. C., Chuang, Y. T., & Cheng, Y. C. (2016). Multiple product admission control in semiconductor manufacturing systems with process queue time (PQT) constraints. *Computers & Industrial Engineering, 99*, 347–363.

Xie, X., Li, J., Swartz, C. H., Dong, Y., & DePriest, P. (2016). Modeling and analysis of ward patient rescue process on the hospital floor. *IEEE Transactions on Automation Science and Engineering, 13*(2), 514–528.

Zantek, P. F., Wright, G. P., & Plante, R. D. (2006). A self-starting procedure for monitoring process quality in multistage manufacturing systems. *IIE Transaction, 38*, 293–308.

Zhang, L., Wang, C., Arinez, J., & Biller, S. (2013). Transient analysis of Bernoulli serial lines: Performance evaluation and system-theoretic properties. *IIE Transactions, 45*(5), 528–543.

Zhao, C., & Li, J. (2014). Analysis and improvement of multi-product assembly systems: An application study at a furniture manufacturing plant. *International Journal of Production Research, 52*(51), 6399–6413.

Zhao, C., Li, J., & Huang, N. (2016). Efficient algorithms for analysis and improvement of flexible manufacturing systems. *IEEE Transactions on Automation Science and Engineering, 13*(1), 105–121.

Zhong, X., Lee, H. K., & Li, J. (2017). From production systems to health care delivery systems: A retrospective look on similarities, difficulties and opportunities. *International Journal of Production Research, 55*(14), 4212–4227.

Zhong, X., Li, J., Bain, P. A., & Musa, A. J. (2016a). Electronic visits in primary care: modeling, analysis, and scheduling policies. *IEEE Transactions on Automation Science and Engineering. 99*, 1–16.

Zhong, X., Li, J., Ertl, S. M., Hassemer, C., & Fiedler, L. (2016b). A system-theoretic approach to modeling and analysis of mammography testing process. *IEEE Transactions on Systems, Man, and Cybernetics: Systems, 46*(1), 126–138.

Zhou, D., Yu, Z., Zhang, H., & Weng, S. (2016). A novel grey prognostic model based on Markov process and grey incidence analysis for energy conversion equipment degradation. *Energy, 109*, 420–429.

Zhu, H., Liu, X., & Chen, Y. F. (2015). Effective inventory control policies with a minimum order quantity and batch ordering. *International Journal of Production Economics, 168*, 21–30.